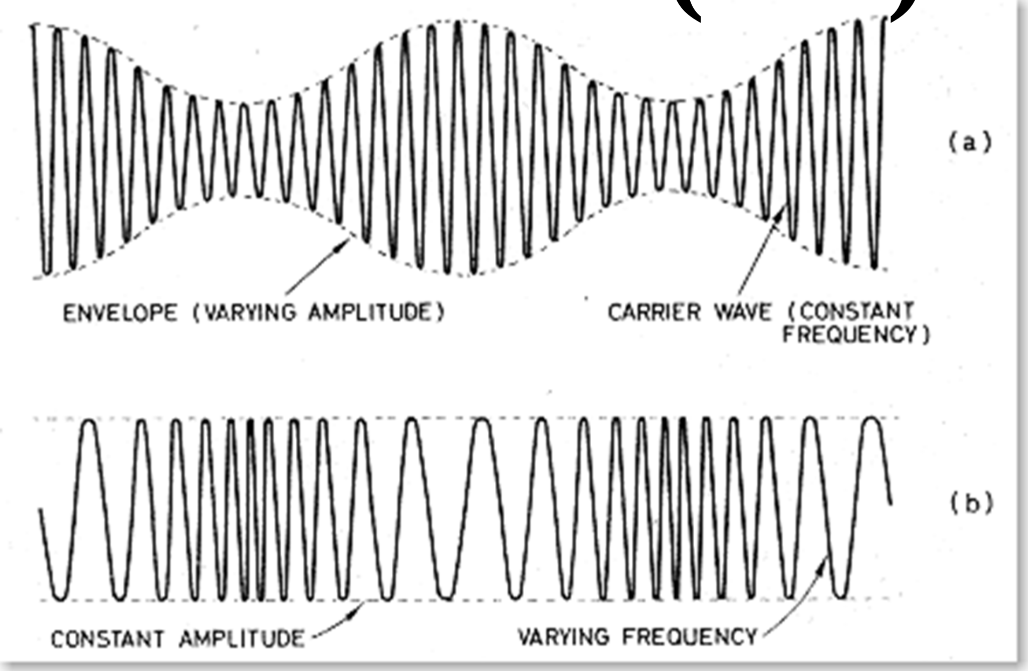
Frequency Modulation (FM)



FM

- Distortion is more likely to impact the amplitude of tx signal than the frequency
- The message signal is represented by variation in the frequency of the carrier
- The amplitude of message signal is recovered from the frequency of the carrier
- The frequency of message signal is recovered from the rate of change in the frequency of the carrier
- FM radio, Audio portion of TV, Point-to-point radio systems

FM

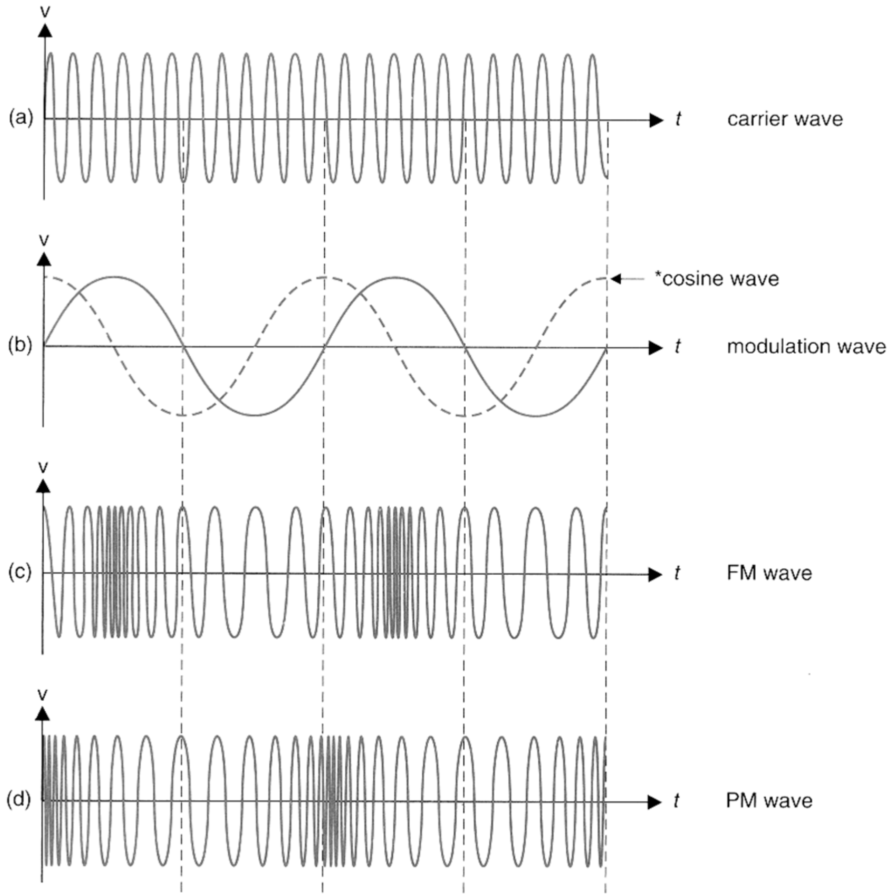


FIGURE 4-1
The FM and PM waveforms for sine-wave modulation: (a) carrier wave; (b) modulation wave; (c) FM wave; (d) PM wave. (*Note:* The derivative of the modulating sine wave is the cosine wave shown by the dotted lines. The PM wave appears to be frequency modulated by the cosine wave.)

FM

$c(t) = A_c \cos w_c t$
$m(t) = A_m \cos w_m t$
$y(t) = A_c \cos \theta(t)$

$d\theta(t)/dt = 2\pi f_c + km(t)$: instantaneous frequency
k: modulation sensitivity
How much instant frequency varies per unit of the input message signal

Angle of y(t): $\theta(t) = \int [w_c + km(t)]dt$
$= w_c t + \int kA_m \cos w_m t \, dt$
$= w_c t + (kA_m/w_m)\sin w_m t$

FM

$y(t) = A_c \cos \theta(t)$
$\quad = A_c \cos [w_c t + (kA_m/w_m)\sin w_m t]$

Modulation index: $\beta = kA_m/w_m = kA_m/2\pi f_m = \Delta f/f_m$

k: radian frequency/volt
Δf: maximum frequency deviation of the carrier by the amplitude of message

FM

Example

$y(t) = A_c \cos [w_c t + \beta \sin w_m t]$

Let $k = 2\pi(10K/sec)/v$, $f_c = 10MHz$, $A_c = 10v$,

$f_m = 4KHz$, $A_m = 2v$

Then

$\beta = \Delta f/f_m = 20K/4K = 5$

$y(t) = 10 \cos [2\pi(10M)t + 5 \sin 2\pi(4K)t]$

Frequency range of the carrier:

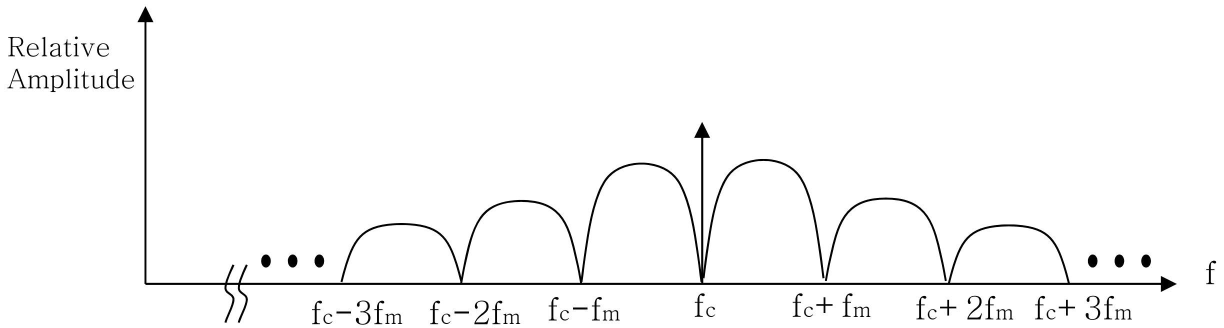
$[f_c - \Delta f,\ f_c + \Delta f] = [10M - 20K,\ 10M + 20K]$

$= [9.98M,\ 10.02M]$

Frequency analysis of FM wave

An FM signal with a carrier frequency w_c and a message frequency w_m contains an infinite number of spectral components at $w_c \pm nw_m$

The amplitude of each sideband is determined by the Bessel function



Frequency analysis of FM wave

TABLE 4-1
Bessel functions of the first kind

		n or order of sidebands															
Modulation Index (m_f)	**Carrier Frequency J_0**	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—	—	—
9.0	−0.09	0.24	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.30	0.21	0.12	0.06	0.03	0.01	—	—	—
10.0	−0.25	0.04	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.31	0.29	0.20	0.12	0.06	0.03	0.01	—	—
12.0	0.05	−0.22	−0.08	0.20	0.18	−0.07	−0.24	−0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01
15.0	−0.01	0.21	0.04	−0.19	−0.12	0.13	0.21	0.03	−0.17	−0.22	−0.09	0.10	0.24	0.28	0.25	0.18	0.12

Source: E. Cambi, *Bessel Functions,* Dover Publications, Inc., New York, N.Y., 1948. Courtesy of the publisher.

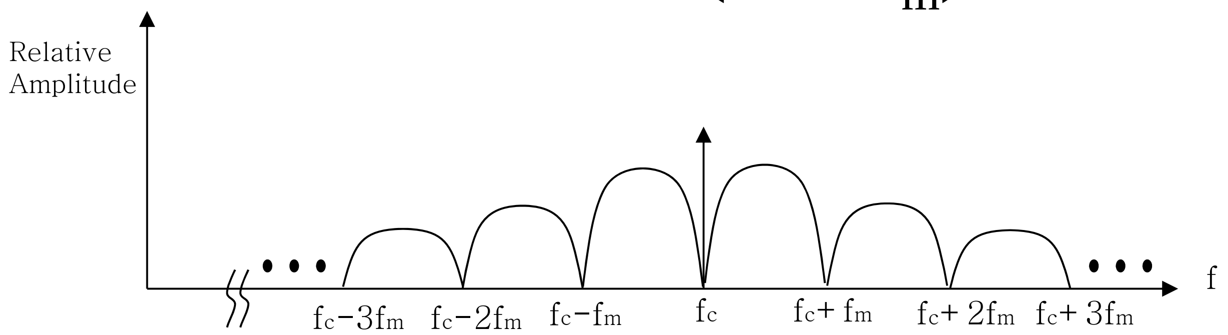
BW requirements for FM

The BW of FM depends on the number of significant sidebands

Carson's Rule: $BW_{FM} = 2\ (\beta+1)\ BW_{BB}$

$= 2\ (\beta+1)\ BW_{BB}$

$= 2\ (\Delta f + f_m)$



Broadcast FM

FM broadcast band: 88 - 108 MHz

100 channels with 200kHz bandwidth

Maximum frequency deviation $\Delta f = 75kHz$

Message frequencies: 50Hz - 15kHz

FM bandwidth requirement

$$\begin{aligned} BW_{FM} &= 2\,(\beta+1)\,BW_{BB} \\ &= 2\,(\Delta f + f_m) \\ &= 180\text{ kHz} \end{aligned}$$

A 10kHz guard band above and below to prevent adjacent channel interference