Dynamic Pricing in the Presence of Strategic Consumers and Oligopolistic Competition

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Figure 2 Relative Revenue Loss for Incorrect Assumption of Myopic Consumer Behavior When It Is Fully Strategic, as a Function of Population Size

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1. Introduction

Dynamic pricing

consumers are able to track prices and available capacity

Explores a model

strategic behavior by both firms and consumers in a unified stochastic dynamic game

oligopolistic firms selling perishable goods

finite segment of consumers
1. Introduction

Numerical experiment measures the effect of strategic behavior

Key conclusion
firms may benefit more from limiting the information available to consumers
2. Basic Model Elements

Decision period: \( t \in \{0, \ldots, T-1\} \)

Firm \( j \) offers a product \( j \) at unit cost \( c_j \) and price \( p_j \)

Integer capacity \( Y_j \geq 0 \rightarrow \) remaining capacity \( y_j \geq 0 \)

\( N \): total \# of consumers

\( s \): \# of consumer segments

\( N_r \): initial segment \( r \) capacity

\((N = \sum_{r=1}^{s} N_r)\)

As consumers acquire items, capacities of firms and market segments are deployed
2. Basic Model Elements

\( a_{trj} \): consumer perception of quality or value of the product

\( \epsilon_{trj} \): random variable with mean zero

\( B_{trj} = a_{trj} + \epsilon_{tr} \): valuation of product j at time t by segment r consumer

\( \beta_r \in [0, 1] \): degree of strategic behavior

- \( \beta_r = 0 \): consumer completely disregards the possibility of future purchase (myopic)
- \( \beta_r = 1 \): consumer values the current purchase the same as at any point in the future (strategic)
3. Modelling Assumptions

(i) Information Availability

Perfect information: Firms and consumers have perfect knowledge of all market information (remaining capacities, market segments, and distributional and parametric characteristics)

Zero information cost: Information relevant for decisions is available at no cost

(ii) Rationality

Perfect foresight: All market participants are sophisticated (they anticipate the strategic behavior and compute resulting event probabilities and expected payoffs)
3. Modelling Assumptions

(iii) Demand Structure

Demand as a counting process:

Aggregate demand from each segment is a counting process with intensity dependent on market conditions such as time, price, and so on.

Each segment process is a sum of independent demand process originating from individual consumers.
3. Modelling Assumptions

Shopping intensity control: Consumers respond to market by controlling shopping intensities

Maximum shopping intensity: A maximum probability of consumer’s eagerness to acquire a particular product. It is same for all consumers.
3. Modelling Assumptions

(iv) Choice Model

Intensity allocation as a choice model: Consumers allocate their shopping intensity among products according to a discrete-choice model with the outside alternative of delay in future.

Valuation are known up to their distribution: Given the consumer’s segment, the state of his/her valuation is known only as a distribution rather than a specific value (applies both to firms and all consumers).
3. Modelling Assumptions

Consumer perceptions are conditional on purchase: After the purchase, the distribution of valuations perceived by the consumers is the conditional valuation distributional given that the purchased product is preferred to any other option. Consumers and firms account for this shift in distributions in their decision making.
4. The Competitive Game

(1) Firm’s Game and Consumer Response

At time $t$

$y = (y_1, \ldots, y_m)$ - remaining capacities of the firms

$n = (n_1, \ldots, n_s)$ - remaining # of consumers across segments

$(y, n)$ - information state of firms’ decision

$(y, n, p)$ - information state of consumer decision
4. The Competitive Game

Move of the firms
\[ p(t, y, n) = (p_1(t, y, n), \ldots, p_m(t, y, n)) \]

Shopping intensity (response of segment \( r \) consumers)
\[ \lambda_r(t, y, n, p) = (\lambda_{r1}(t, y, n, p), \ldots, \lambda_{rm}(t, y, n, p), \ r = 1, \ldots, s) \]

Probability that a unit of product \( j \) is sold to and individual consumer of segment \( r \)
\[ \lambda_{rj}(t, y, n, p) \]

Probability of no sale
\[ 1 - \sum_{r,j} n_r \lambda_{rj}(t, y, n, p) \]
4. The Competitive Game

Maximum shopping intensity

\[ \lambda_{rj}(t,y,n,p) \leq \bar{\lambda} \text{ for all } t, r, j \text{ and } (y,n,p) \]

Size of discretization

\[ \bar{\lambda}mN \text{ (sufficiently small compared to 1: probability of more than one purchase in a given decision period is negligible)} \]

Probability of segment r consumer choosing product j at given price \( p = (p_1, \ldots, p_m) \)

\[ P(B_{trj} - p_j = \max_{j=1,\ldots,m} (B_{trj'} - p_{j'})) \]
4. The Competitive Game

Expected surplus of a representative myopic consumer in segment $r$

$$E_{B_{tr}}[\max_{j'=1,\ldots,m} (B_{trj'} - p_{j'})]$$

$$= \sum_{j=1}^{m} \int_{b_j-p_j \geq b_{j'}, -p_{j'}, j' \neq j} (b_j - p_j) f_{tr}(b) \, db$$

Certainty equivalent of a future purchase, at time $t$, information state $(y, n, p)$ as evaluated by segment $r$ consumer

$$Q_r(t, y, n, p) : \text{certainty equivalent of a future purchase}$$

(value of the option of delaying purchase)
4. The Competitive Game

Segment $r$ strategic consumer chooses product $j$ with probability $P(B_{tr} \in A_{rj}(t, y, n, p))$

where

$$A_{trj}(t, y, n, p) = \{ b \in \mathbb{R}^m : b_j - p_j = \max_{j'=1,\ldots,m} (b_{j'} - p_{j'}) \}$$

$\forall b_j - p_j \geq Q_r(t, y, n, p)$

Proposition 1: The response of any of the $n_r$ segment $r = 1, \ldots, s$ consumer in info state $(y, n, p)$ are

$$\lambda_{rj}(t, y, n, p) = \overline{\lambda}(B_{tr} \in A_{rj}(t, y, n, p))$$
4. The Competitive Game

(2) Expected Utility of a Future Purchase

Expected utility of a segment $r$ consumer in info state $(y, n)$ and $(y, n, p)$ at time $t$

$U_r(t, y, n)$ and $U_r(t, y, n, p)$

At the end of planning horizon

consumers have the expected utility of 0

$U_r(T, y, n) = 0$ for all $r$ and $(y, n)$

If the firms run out of capacity, expected utilities are 0

$U_r(t, 0, n) = 0$ for all $t, r, n$
4. The Competitive Game

\( p^*(t, y, n) \): equilibrium strategy profile

\[ U_r(t, y, n) = E[U_r(t, y, n, p^*(t, y, n))] \]

To complete recursion, need to find \( U_r(t, y, n, p) \) as a function of expected utilities at time \( t+1 \)

Given a \( k \) dimensional vector \( z \) and by replacing the \( l \)th component \( z \) with \( \hat{z} \) we can obtain \( (\hat{z}, z_{-l}) \)
4. The Competitive Game

Proposition 2: The expected utility of a segment $r$ consumer in information state $(y, n, p)$ is

$$U_r(t, y, n, p) = \lambda \sum_{j=1}^{m} \int_{b \in A_{rj}(t,y,n,p)} (b_j - p_j - \beta_r U_r(t+1, y, n)f_{tr}(b))db$$

$$- \beta_r \sum_{r' \neq r} (n_{r'} - I(r' = r)) \lambda_{r'}(t, y, n, p)^T \Delta_{r'}U_r(t+1, y, n) + \beta_r U_r(t+1, y, n)$$

where

$$\Delta_{r'}U_r(t+1, y, n), r = 1, ..., s$$ is a vector of terms

$$\Delta_{r'} U_r(t + 1, y, n) = U_r(t + 1, y, n) - U_r(t + 1, (y_j - 1, y_{-j}), (n_{r'}, -1, n_{-r'})), j = 1, ..., m$$
4. The Competitive Game

Value of an Explicit Delay

For a segment $r$ consumer at time $t$ in state $(y, n, p)$ the value of the option of explicitly delaying the purchase

$$Q_r(t, y, n, p) = \beta_r U_r(t+1, y, n)$$
4. The Competitive Game

(3) Segment Demand, Firms’ Payoffs, and Equilibrium

Corollary 1: Under conditions of Proposition 2, demand intensity for product j from segment r in information state \((y, n, p)\) is

\[
D_{rj}(t, y, n, p) = n\lambda P(B_{tr} \in A_{rj}(t, y, n, p))
\]
4. The Competitive Game

Equilibrium expected payoff

\[ R_j(t, y, p) \]

At the end of planning horizon

\[ R_j(T, y, n) = 0 \text{ for all } (y, n) \]

If the firms run out of capacity

\[ R_j(t, 0, n) = 0 \text{ for all } (y, n) \text{ such that } y_j = 0 \text{ or } n = 0 \]
4. The Competitive Game

Expected future profit of firm $j$

$$R_j(t, y, n, p)$$

$$= \sum_{r=1}^{s} D_{rj}(t, y, n, p)$$

$$\cdot (p_j + R_j \left( t + 1, (y_j - 1, y_{-j}), (n_r - 1, n_{-r}) \right) - c_j)$$

$$+ \sum_{r=1}^{s} \sum_{j' \neq j} D_{rj'}(t, y, n, p)$$

$$\cdot R_j(t + 1, (y_{j'}, -1, y_{-j'}), (n_r - 1, n_{-r}))$$

$$+(1 - \sum_{r=1}^{s} \sum_{j'=1}^{m} D_{rj'}(t, y, n, p) R(t + 1, y, n)$$
4. The Competitive Game

We can rewrite $R_j(t, y, n, p)$

$$R_j(t, y, n, p)$$

$$= \sum_{r=1}^{s} D_{rj}(t, y, n, p)(p_j - \Delta_{rj} R_j(t + 1, y, n) - c_j)$$

$$- \sum_{r=1}^{s} \sum_{j' \neq j} D_{rj'}(t, y, n, p) \Delta_{rj'} R_j(t + 1, y, n))$$

$$+ R_j(t + 1, y, n)$$

where

$$\Delta_{rj'} R_j(t + 1, y, n) = R_j(t + 1, y, n) - R_j(t + 1, (y_{j'} - 1, y_{-j'}), (n_r - 1, n_{-r}))$$
4. The Competitive Game

Assumptions

(A) Bounded Prices
The prices used are bounded by a sufficiently large constant $p$

(B) Tie-breaker Mechanism
There is a mechanism that selects an equilibrium to be implemented by the firms if $G(t, y, n)$ has multiple equilibria. This mechanism ensures that equilibrium payoffs in $G(t, y, n)$ are uniquely defined
4. The Competitive Game

Theorem 1

Under assumptions (A) and (B), there exists a Markov-perfect equilibrium in mixed strategies.
5. Generalization of the Choice Model

Generalized maximum shopping intensity
\[
\|x\|_q \leq \lambda, \text{ where } \|x\|_q \text{ is a } q - \text{ norm of } x
\]

\(q=1\) (specific choice)
\[
\|x\|_1 = \sum_{j=1}^{m} x_j
\]

\(q=\infty\) (multiple choice)
\[
\|x\|_{\infty} = \max_{j=1,\ldots,m} |x_j|
\]
5. Generalization of the Choice Model

Averaging Behavior

assume that a strategic consumer behaves as follows:
first, a consumer determines his/her optimal shopping intensity
second, consumer averages those intensities
5. Generalization of the Choice Model

Proposition 3

decision period $t$, suppose that there exists a unique equilibrium in all subsequent periods.

Then the equilibrium response of any of the $n_r$ consumers in segment $r$, info state $(y, n, p)$

$$\lambda_{rj}(t, y, n, p) = E[x^*_{rj}(t, y, n, p, B_{tr})]$$

$(x^*_{rj}(t, y, n, p, B_{tr})$: optimal shopping intensity $)$
5. Generalization of the Choice Model

for \( q=\infty \) (multiple)

\[
x^*_{trj}(t, y, n, p, B_{tr}) = \bar{\lambda} I(B_{trj} - p_j \geq \beta_r U_r(t+1, y, n)),
\]

\[
\text{for } j = 1, \ldots, m
\]

for \( q=1 \) (specific)

\[
x^*_{trj}(t, y, n, p, B_{tr}) = \bar{\lambda} I(B_{tr} - p_j \in A_{rj}(t, y, n, p)),
\]

\[
\text{for } j = 1, \ldots, m
\]
5. Generalization of the Choice Model

Corollary 2

Let \( q = \infty \) and consider decision time period \( t \), and suppose that there exists a unique equilibrium in all subsequent time periods. Then, the demand intensity for product \( j \) from segment \( r \) in info state \( (y, n, p) \) is

\[
D_{rj}(t, y, n, p) = nr \bar{\lambda} P(B_{trj} - p_j \geq \beta_r U_r(t+1, y, n))
\]

Corollary 3

Under conditions of Corollary 2, expected future profit of firm \( j \) is separable in the components of \( p \) for all \( j = 1, \ldots, m \)
5. Generalization of the Choice Model

Theorem 2

For the case $q = \infty$ and under assumptions (A) and (B), there exists a Markov-perfect equilibrium in pure strategies.

Theorem 3

For the case $q = \infty$, $s = 1$, and for all $t$, the distribution of $B_{tr}$ satisfies the assumptions of

(C) Logconcavity: Marginal density of $f_{trj}(b)$ of each component of $B_{tr}$ is logconcave, and

(D) Regularity: The marginal distribution of each component of $B_{tr}$ satisfies the regularity condition $\lim_{b \to \infty} bP(B_{trj} \geq b) = 0$
6. Managerial Insight from Numerical Illustrations

When unit costs are negligible for each $t$ and $r$, the $B_{trj}$, $j=1,\ldots,m$ are independent and identically distributed according to the exponential distribution with mean $a_{rj}$

$$f_{tr}(b_j) \frac{1}{a_{rj}} e^{-b/a_{rj}}$$

Some aspects of real markets can affect the results-information or waiting cost
Consumer arrivals and departures without a purchase
Rationing the product sales by firms
Limited information availability
6. Managerial Insight from Numerical Illustrations

Numerical experiments to study the effects of strategic behavior on equilibrium and the effects of deviation from equilibrium.

Percentage of revenue difference- effect of strategic behavior as the percentage difference in revenues between the strategic and purely myopic cases for each firm relative to the myopic case.
6. Managerial Insight from Numerical Illustrations

(1) Effects of strategic behavior

Considering 2 market segments

\[ \beta = 0 \ & \beta = 1 \] (otherwise identical)

Difference in percentage of revenues is much smaller in fully strategic consumers.

Strategic effect is strongest when firms divide the market until \( N > 50 \) (competition among consumers for a limited product supply reduces their expected utility)

When, \( N < 40 \), it has smaller effect

Possible explanation- lower level of prices and revenues because of more intense competition between the companies
6. Managerial Insight from Numerical Illustrations

(2) Interplay of strategic behavior

Comparing symmetric equilibria under the multiple consumer choice assumption

Observation: strategic consumer behavior leads to lower revenues, and this effect is aggravated by the presence of competitors.
6. Managerial Insight from Numerical Illustrations

In competitive situations, the largest differences in revenue occur when the firms divide the market.

Reason- when consumers are myopic and the market size is increased proportionally with the number of firms there is a noticeable increase in revenues.

Increases are made possible by the stochastic nature of the market because the firms have more opportunities to set higher prices when they face a market of proportionally larger size despite the presence of competitors.

Dependency of result
- On the stochastic nature of the market
- Information availability
6. Managerial Insight from Numerical Illustrations

(3) Losses from Deviating from equilibrium by wrongly assuming that the consumers are myopic

Examining three situations

![Figure 2](image.png)

**Relative Revenue Loss for Incorrect Assumption of Myopic Consumer Behavior When It Is Fully Strategic, as a Function of Population Size**
6. Managerial Insight from Numerical Illustrations

Result

Strong firm is generally less affected by deviation than the weak firm or the firm in symmetric scenario.
The losses decrease when N increase and there is more intense competition between the consumers.
7. Conclusion

Most restrictive assumptions
   “information availability”

To improve the situation
   Firms to learn characteristics of the market as time progresses and continuously adjust the optimal pricing policy
   Incorporation of consumer learning will provide a more realistic representation of consumer behavior
References

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