Social Networks, Information Acquisition, and Asset Prices


I.W.Kim
M.K.Choi
Index

- Introduction
- The model
- Financial Market Equilibrium: Exogenous $\mu$
  - Equilibrium Characterization
  - Implications
- Information Market Equilibrium: Endogenous $\mu^*$
  - Equilibrium Characterization
  - Implications
- Concluding Remarks
- References
Introduction

- A central topic in the studies of financial markets
  - how much information will be acquired about stock fundamentals
  - how much the stock price will reflect investors’ diverse information

- Using *rational expectations model*
  - to study the incentives to acquire costly *information* on the value of a stock.
  - i) the people in the economy make choices based on their rational outlook, available information and past experiences.
    - ii) the current expectations in the economy are equivalent to what the future state of the economy will be.

Ex) If a company believes that the price for its product will be higher in the future, it will stop or slow production until the price rises. In sum, the producer believes that the price will rise in the future, makes a rational decision to slow production. [1]
Introduction

- How information disseminates through agents in financial markets
  - *Information sharing* with peers via social networks plays an important role for investment decision making such as stock market participation and portfolio choices [2], [3], [4], [5], [6]
  - Information transfers via social interactions, word-of-mouth communication among friends and neighbors, and shared education networks, etc.
  - Social networks play a direct role in facilitating the *price discovery process* [7]
    - Price discovery: A method of determining the price for a specific commodity or security through basic supply and demand factors related to the market. [8]
  - As a result, researchers have started to examine *how market information efficiency depends on the structure of a social network* [9], [10]
In this paper,

- Rational expectations equilibrium model of a competitive market in which heterogeneous traders can learn about a risky asset’s payoff from three sources: market price, costly information acquisition, and communications via social network.

- The network structure is taken to be exogenous.

- When a trader decides whether to incur a cost to acquire information, he takes into consideration the expected learning via social communications.

- Traders’ conjectures about how much information is revealed through price are fulfilled by their own acquisition activities.
Introduction

Goals

To study information acquisition in a financial market with social networks

To examine how network connectedness affects equilibrium market outcomes: 1. price informativeness 2. cost of capital 3. liquidity 4. trading volume.

To investigate the interactions between information acquisition and communication of information via social networks by comparing market outcomes when information is endogenously acquired with a cost versus when information is exogenously given.

This paper shows,

how a social network affects market outcomes depends on whether private information is exogenously given or endogenously acquired at a cost.
Introduction

- Information is exogenous Vs. endogenous
  - Exogenous information
    - 2 positive effects
      - Information sharing enlarges everyone’s information set and thus the precision of a stock’s payoff conditional on each trader’s information set is higher.
      - Sharing information among more friends causes more information to be impounded into the price, thereby improving informational efficiency.
    - As a consequence, when information is exogenous, more social communications increase the average trading aggressiveness of traders, which, in turn, lowers cost of capital, increases stock liquidity and trading volume.
Introduction

- Information is exogenous Vs. endogenous

- Endogenous information
  - 2 positive effects in exogenous information + negative effect
    - In anticipation of learning from informed friends and more informative market price, *traders would have less incentive to incur a cost* and acquire information on their own, thereby *reducing the total amount of information produced* in the economy.

- ‘Information is exogenous Vs. endogenous’ depends on the cost of acquiring information
  - the cost of acquiring information is *low*,
    - most traders choose to collect information even in the presence of social communications, so that the *positive effects dominate*.
  - the cost of acquiring information is *high*,
    - fewer people choose to acquire information at a cost, which lowers traders’ trading aggressiveness, raises cost of capital, and harms liquidity and volume.
    - The *negative effects dominate*.
Introduction

- the relation between network connectedness and the economic outcome depends on the information acquisition cost
  - The economic outcome variables such as the cost of capital.
  - the cost of acquiring information is low
    - the fraction of informed traders is not sensitive to network connectedness,
    - the cost of capital is significantly negatively related to network connectedness.
  - the cost of acquiring information is high
    - more social communications lead to a smaller amount of information production
    - the cost of capital is significantly positively related to network connectedness.

- Similar predictions apply to the relation between price efficiency and the amount of social communications among investors.
Introduction

Several recent theoretical studies show that social communications improve market efficiency.[11][12][13]

- In all of these studies, information is exogenous.
- This paper complement these studies by showing that when information is endogenous,
  - social communications can affect economic outcomes in a way that is opposite to the exogenous information case.
  - also derive implications of social network for cost of capital and market liquidity
The Model

- **Using pure exchange**
  - Pure exchange model [14]
    - What if one person who possesses one type of good (say apples) meets up with another person who possesses another type of good (say oranges)?
    - What could we say about two people trading apples for oranges?
  - One period and two assets
    - a riskfree asset (bond)
      - is assumed to pay off in one unit of the single consumption good and has a constant value of 1
    - a risky asset (stock)
      - Has an uncertain payoff at the end of the period, denoted $\tilde{v}$
      - $\tilde{v} \sim N(\bar{v}, 1/\rho_v)$ with $\bar{v} > 0$ and $\rho_v > 0$
The Model

- Traders
  - [0, 1] continuum of rational traders who have constant-absolute-risk-aversion (CARA) utility with a risk aversion coefficient $\gamma > 0$
  - They are endowed with only riskfree asset at the beginning of the period
  - Before trade, they choose whether to spend an amount $c > 0$ to become informed
  - Let $\mu$ be the (endogenous) fraction of informed traders
  - Informed traders receive diverse private signals regarding the stock payoff.
  - A competitive financial market then opens and traders (informed, uninformed and noise traders) trade.
  - Noise traders supply $\bar{x}$ units of stock per capita to the market.
    - $\bar{x} \sim N(x, 1/\rho_x)$ with $x > 0$ and $\rho_x > 0$
The Model

- Information network
  - In addition to acquiring information at a cost and learning from price, traders can freely communicate to others that are connected to them in the network.

- the islands-connections model
  - This cost structure captures *heterogeneity in link costs* in a simple manner: *agents are grouped on “islands”, and costs of connection are relatively low within an island and relatively high across islands*. This cost structure, together with the indirect benefits structure of the connections model, generates the small-world characteristics[14]

There are a total mass of $1/N$ groups (islands) in the economy, each of which has $N \geq 1$ traders.

  - Each member of the group is independently *randomly sampled* from the total rational traders population.

  - Thus, the fraction of groups having $m$ informed traders and $(N - m)$ uninformed traders is given by the binomial coefficient:

$$\pi_m^N \triangleq \frac{N!}{m! (N - m)!} \mu^m (1 - \mu)^{N-m}$$
The Model

Information network

- Within any group every trader is connected to all other traders in the group, but there are no links across groups
  - refer to traders in the same group as “friends”
  - refer \( N \) to network connectedness

Information structure

- Informed trader \( i \) in a group \( g \) (after paying a cost \( c \)) observes a signal:
  \[
  \tilde{s}_{i,g} = \tilde{v} + \tilde{\varepsilon}_{i,g}, \text{ with } \tilde{\varepsilon}_{i,g} \sim N(0, 1/\rho_\varepsilon) \text{ and } \rho_\varepsilon > 0.
  \]
  - Informed traders share their signals with other members of the same group

- To capture the fact that some private information may be difficult to share (e.g., due to communication barriers), we assume that other traders in group \( g \) receive a noisy signal from trader \( i \) as follows:
  \[
  \hat{y}_{i,g} = \tilde{s}_{i,g} + \tilde{\eta}_{i,g}, \text{ with } \tilde{\eta}_{i,g} \sim N(0, 1/\rho_\eta) \text{ and } \rho_\eta \geq 0.
  \]
The Model

Figure 1: Timeline

- Traders receive endowments
- Social networks are formed
- \( \mu \) fraction of traders observe signals at a cost \( c \):
  \[ \tilde{s}_{i,g} = \tilde{v} + \tilde{\epsilon}_{i,g} \]
- Each informed trader broadcasts his signal to his friends:
  \[ \tilde{y}_{i,g} = \tilde{s}_{i,g} + \tilde{\eta}_{i,g} \]
- Rational traders trade bonds and stocks
- Noise traders supply random \( \bar{x} \) shares of stocks
- Stock payoff \( \tilde{v} \) realized
- Rational traders consume

This figure plots the order of events.
The Model

- assuming a continuum of traders in the whole economy and a finite number of traders in each group
  - to capture the fact that the number of friends (social connections) each trader has is much smaller compared to the number of market participants.
  - This assumption implies that no traders have price impacts in our economy so our model avoids
    - the “non-truthful” reporting problem
    - the “schizophrenia” problem
  - to ensure the existence of analytical solutions for any network connectedness $N$
    - as the noises contained in the private signals of informed traders cancel out in the price function.
Financial Market Equilibrium: Exogenous $\mu$

- **Equilibrium characterization**
  - solves the equilibrium stock price, taking as given the fraction $\mu$ of informed traders
    - Suppose traders conjecture the price function
      \[ \hat{p} = \alpha_0 + \alpha_v \hat{v} - \alpha_x \hat{x}. \]
    - The information contained in the price is equivalent to the following signal
      \[ \hat{\theta} = \frac{\hat{p} - \alpha_0 + \alpha_x \bar{x}}{\alpha_v} = \hat{v} - \frac{\alpha_x}{\alpha_v} (\hat{x} - \bar{x}) \]
Equilibrium characterization

Proposition I

- There exists a partially revealing rational expectations equilibrium in the financial market, with price function

\[ \tilde{p} = \alpha_0 + \alpha_v \tilde{v} - \alpha_x \tilde{x} \]

where

\[ \alpha_0 = \frac{\rho_y \tilde{v} + (\alpha_v/\alpha_x) \rho_x \tilde{x}}{\rho_v + \rho_\theta + \mu \rho_\varepsilon + \mu (N - 1) \rho_y}, \]

\[ \alpha_v = \frac{\rho_v + \mu \rho_\varepsilon + \mu (N - 1) \rho_y}{\rho_v + \rho_\theta + \mu \rho_\varepsilon + \mu (N - 1) \rho_y}, \]

\[ \alpha_x = \frac{(\alpha_v/\alpha_x) \rho_x + \gamma}{\rho_v + \rho_\theta + \mu \rho_\varepsilon + \mu (N - 1) \rho_y}, \]

where

\[ (\alpha_v/\alpha_x) = \frac{\mu \rho_\varepsilon + \mu (N - 1) \rho_y}{\gamma} \] and \[ \rho_\theta = \frac{\mu^2 [\rho_\varepsilon + (N - 1) \rho_y]^2 \rho_x}{\gamma^2} \]
Financial Market Equilibrium: Exogenous $\mu$

**Implications**

- The implications of social communications for aggregate outcomes in economies with an exogenous information structure (a fixed $\mu$) by examining how varying $N$ changes market efficiency, cost of capital, liquidity and trading volume.
  
  - The role of $N$ in determining market outcomes can be well understood by looking at its impact on market efficiency, because its impact on other aggregate variables is closely linked to its impact on market efficiency.

- By proposition I,
  
  - Increasing the networks connectedness $N$ has a positive effect on market efficiency $\rho_\theta$ in a setting with exogenous information
    
    $$\frac{\partial \rho_\theta}{\partial N} = \frac{2\mu^2 [\rho_x + (N-1)\rho_y] \rho_x \rho_y}{\gamma^2} > 0 \quad (\mu > 0)$$
  
  - Sharing information among more friends causes more information to be impounded into the price, thereby improving informational efficiency.
Financial Market Equilibrium: Exogenous $\mu$

- **Implications**

- Defining the cost of capital as
  \[
  E(\tilde{v} - \hat{p}) = \frac{\gamma \tilde{x}}{\rho_v + \rho_\theta + \mu \rho_\epsilon + \mu (N - 1) \rho_y}
  \]

  - For a given $\mu > 0$, increasing $N$ will lower the cost of capital, since
    \[
    \frac{\partial \left( \rho_v + \rho_\theta + \mu \rho_\epsilon + \mu (N - 1) \rho_y \right)}{\partial N} = \frac{\partial \rho_\epsilon}{\partial N} + \mu \rho_y > 0
    \]

  - First, more information sharing via social networks makes everyone better informed and less uncertain about the stock payoff.
    - This increases the aggregate demand for the stock. Thus, the equilibrium price is higher and the cost of capital is lower.

  - Second, there is an indirect effect on the cost of capital through the revelation of information by the stock price
    - More information sharing via social networks causes the price system to reveal more information to all traders, which makes the stock less risky and further reduces the cost of capital.

  - These two effects combine to make social communications reduce the cost of capital.
Financial Market Equilibrium: Exogenous $\mu$

Implications

- Liquidity of the market is measured by $1/\alpha_x$
  - $\alpha_x$: to capture the market depth (how much a unit of non-information-driven stock transaction can move prices (i.e., by the price equation, we have $\frac{\partial \bar{\sigma}}{\partial \bar{x}} = -\alpha_x$)
  - By proposition I,
    $$\frac{1}{\alpha_x} = \frac{\rho_v + \rho_x (\alpha_v/\alpha_x)^2 + \gamma (\alpha_v/\alpha_x)}{\rho_x (\alpha_v/\alpha_x) + \gamma}$$
  - the effect of N on $1/\alpha_x$ works through its effect on the price-informativeness measure, $\alpha_v/\alpha_x$ or equivalently on $\rho_\theta$, since $\rho_\theta = (\alpha_v/\alpha_x)^2 \rho_x$
  - An increase of $\alpha_v/\alpha_x$ has two effects on liquidity measure $1/\alpha_x$
    - it increases the average trading aggressiveness of rational traders (i.e., it increases the numerator in equation)
    - because each trader can learn more information from prices. This causes the aggregate demand of rational traders to be more sensitive to prices,
    - as a result, an extra unit of liquidity supply (i.e., an increase in $\bar{x}$) tends to move equilibrium prices by a smaller amount, leading to a higher liquidity.
Implications

- Liquidity of the market is measured by $1/\alpha_x$
  
  - By proposition I,
    \[
    \frac{1}{\alpha_x} = \frac{\rho_v + \rho_x (\alpha_v/\alpha_x)^2 + \gamma (\alpha_v/\alpha_x)}{\rho_x (\alpha_v/\alpha_x) + \gamma}
    \]

  - Direct computation shows that
    \[
    \frac{\partial (1/\alpha_x)}{\partial (\alpha_v/\alpha_x)} = \frac{\left[\rho_x (\alpha_v/\alpha_x) + \gamma\right]^2 - \rho_x \rho_v}{\left[\rho_x (\alpha_v/\alpha_x) + \gamma\right]^2}
    \]

Thus, $\frac{\partial (1/\alpha_x)}{\partial (\alpha_v/\alpha_x)} > 0$ if and only if

\[
\frac{\alpha_v}{\alpha_x} > \frac{\sqrt{\rho_v} - \gamma}{\rho_x}
\]  

(20)

Since by Proposition I, $\frac{\partial (\alpha_v/\alpha_x)}{\partial N} = \frac{\mu \rho_y}{\gamma} > 0$ for a fixed $\mu > 0$ then,

\[
\frac{\partial (1/\alpha_x)}{\partial N} = -\frac{\partial (1/\alpha_x)}{\partial (\alpha_v/\alpha_x)} \frac{\mu \rho_y}{\gamma} > 0
\]

if and only if condition (20) is satisfied, or the following condition is satisfied

\[
\mu \left[\frac{\rho_v + (N - 1) \rho_y}{\gamma}\right] > \sqrt{\frac{\rho_v}{\rho_x} - \frac{\gamma}{\rho_x}}
\]  

(21)
Financial Market Equilibrium: Exogenous $\mu$

- Implications
  - Proposition II
    - When the information structure is exogenous, increasing the connectedness of networks will improve market efficiency and lower cost of capital. That is, for a fixed $\mu > 0$, $\frac{\partial \rho_\theta}{\partial N} > 0$ and $\frac{\partial E(\tilde{\psi} - \tilde{\psi})}{\partial N} < 0$. In addition, increasing the connectedness of networks will improve liquidity if and only if condition (21) is satisfied. That is, for a fixed $\mu > 0$, $\frac{\partial (1/\alpha_\chi)}{\partial N} > 0$ if and only if (21) holds.
Financial Market Equilibrium: Exogenous $\mu$

Figure 2: Implications of Networks in Economies with Exogenous Information

(a) mkt eff  (b) E($v-p$)  (c) liquidity  (d) volume

how market efficiency ($\rho_\theta$), cost of capital($E(\tilde{v} - \tilde{p})$), liquidity($1/\alpha_\chi$) and volume($E(\tilde{q})$) vary with connectedness $N$ of networks in economies with exogenous information.
Equilibrium characterization

- $\mu$ fraction of traders have already purchased the private signal, the expected net benefit of the information to a potential purchaser is

\[
B(\mu; N) = \frac{\sum_{I_g=0}^{N-1} \left( \prod_{I_g=0}^{N-1} \left( \frac{1}{\sqrt{\rho_v + \rho_g + I_g \rho_y}} \right) \right)}{\sum_{I_g=0}^{N-1} \left( \prod_{I_g=0}^{N-1} \left( \frac{1}{\sqrt{\rho_v + \rho_g + \rho_e + I_g \rho_y}} \right) \right)} - e^{\gamma c}
\]

- $B(0; N) \leq 0$: potential buyer doesn’t benefit from becoming informed when no traders informed $\rightarrow \mu^* = 0$
- $B(1; N) \geq 0$: potential buyer is strictly better off by being informed when all other traders are also informed $\rightarrow \mu^* = 1$
- $B(\mu^*; N) = 0$: every potential buyer is indifferent to becoming informed versus remaining uninformed
Mathematical process

- The numerator and denominator of the first term are taking expectations with respect to a binomial distribution

\[
B(\mu; N) = \frac{E_{I_g}^{N-1} \left( \frac{1}{\sqrt{\rho_v + \rho_\theta + I_g \rho_y}} \right)}{E_{I_g}^{N-1} \left( \frac{1}{\sqrt{\rho_v + \rho_\theta + \rho_\varepsilon + I_g \rho_y}} \right)} - e^{\gamma c}
\]

- Take the first-order approximation

\[
E_{I_g}^{N-1} \left( \frac{1}{\sqrt{\rho_v + \rho_\theta + I_g \rho_y}} \right) \approx \frac{1}{\sqrt{\rho_v + \rho_\theta + E_{I_g}^{N-1} (I_g) \rho_y}};
\]

\[
E_{I_g}^{N-1} \left( \frac{1}{\sqrt{\rho_v + \rho_\theta + \rho_\varepsilon + I_g \rho_y}} \right) \approx \frac{1}{\sqrt{\rho_v + \rho_\theta + \rho_\varepsilon + E_{I_g}^{N-1} (I_g) \rho_y}}.
\]
Information Market Equilibrium: Endogenous $\mu$

- Mathematical process

  - So we have

  $$B(\mu; N) \approx \hat{B}(\mu; N) = \frac{1}{\sqrt{\rho_v + \rho_\theta + E_{I_g}^{N-1}(I_g)\rho_y}} - e^{\gamma c} = \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v + \rho_\theta + (N - 1)\mu\rho_y}} - e^{\gamma c}.$$  

- The expression of $B(\mu; N)$

$$\hat{B}(\mu; N) = \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v + \rho_\theta + (N - 1)\mu\rho_y}} - e^{\gamma c}$$
This equation describes the impact of social communication on the learning benefit:

$$\hat{B}(\mu; N) = \sqrt{1 + \frac{\rho_\varepsilon}{\rho_v} + \frac{\rho_\theta}{\rho_\theta} + (N - 1) \mu \rho_y} - e^{\gamma c}$$

- Two effects are at work here:
  - $(N-1)\mu$ is average number of informed friends and $\rho_y$ is the precision of the signal passed on from an informed friend.
  - Traders also learn from market price, the improved market efficiency will affect the learning incentive, and this channel is reflected by the term $\rho_\theta$.
There are two important properties of function $B$

- First, for a fixed $N$, function $B$ decreases with the fraction of informed traders $\mu$
  - The expected gain to be informed is small when there are already many informed traders
  - Given a social network, as more traders become informed, an uninformed trader expects to glean more information from both friends and prices

- Second, for a fixed $\mu$, function $B$ decreases with network connectedness $N$
  - Increasing the connectedness $N$ of networks will shift downward the function $B$
  - Social communications give traders a chance to learn from each other and also cause prices to reveal more information
  - The incentive to become informed is reduced, resulting in a smaller fraction of informed traders

\[
\hat{B}(\mu; N) = \sqrt{1 + \frac{\rho_y}{\rho_x + \mu^2 [\rho_y + (N - 1) \rho_y]^2 \rho_x / \gamma^2 + (N - 1) \mu \rho_y} - e^\gamma}
\]
Information Market Equilibrium: Endogenous $\mu$

- Learning benefit and information market equilibrium
Implications

The implications of social communications for economic outcomes could be the opposite of those under exogenous information.

- Setting $B(\mu^*, N) = 0$

$$\sqrt{1 + \frac{\rho_y}{\rho_u + \rho_{g} + (N - 1) \mu^* \rho_y}} = e^{\gamma_c}$$

- It is equivalent to

$$\rho_v + \rho_{g} + \mu^* (N - 1) \rho_y = \frac{\rho_y}{e^{2\gamma_c} - 1}.$$

- Increasing $N$ will increase the expected precision $\mu^*(N-1)\rho_y$ of signals shared with friends.

- Then to maintain equation, market efficiency $\rho^*_{\theta}$ has to decrease.

- Therefore increasing $N$ will reduce market efficiency when information is endogenous.
Implications

The average risk faced by traders increase with network connectedness. Thus more social communications will increase the cost of capital.

- Average conditional precision of stock payoff $v$ across all traders is
  \[
  \rho_v + \rho^*_z + \mu^* \rho_z + \mu^* (N-1) \rho_y.
  \]

- By previous equation, we have
  \[
  \rho_v + \rho^*_z + \mu^* \rho_z + \mu^* (N-1) \rho_y = \mu^* \rho_z + \frac{\rho_z}{e^{2\gamma e} - 1}
  \]

- Because, increasing $N$ will decrease $\mu^*$, which leads to a higher average risk faced by traders, and thus a larger cost of capital.
New Empirical Predictions

Distinguish Two cases

• First, When the cost of acquiring cost is low, the fraction of informed traders is not sensitive to network connectedness, and thus can be regarded as exogenous, In this case the model generates the same qualitative prediction as the exogenous information case: Cost of capital decreases with network connectedness

• Second, when the cost of acquiring information is sufficiently high, cost of capital increases with network connectedness
New Empirical Predictions

Prediction1

- For firms with low information acquisition cost, the cost of capital is significantly negatively related to network connectedness
- For firms with high information acquisitions cost, the cost of capital is significantly positively related to network connectedness

Prediction2

- For firms with low information acquisition cost, market efficiency is higher when there are more social communications among their investors
- For firms with high information acquisition cost, market efficiency is lower when there are more social communications among their investors
We analyze a rational equilibrium model of private information to study the implications of social communications for financial markets.

- When the fraction of informed investors is fixed exogenously, social communications improve market efficiency, reduce the cost of capital, tend to increase liquidity and trading volume.

- Social communications also reduce investors’ incentive to acquire information and this have a negative effect on market efficiency.
Reference


Reference
