Dynamic Pricing Competition with Strategic Customers under Vertical Product Differentiation

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We consider dynamic pricing competition between two firms offering vertically differentiated products to strategic customers who are inter-temporal utility maximizers. Our model can be viewed as a competitive version of the classical model of Besanko and Winston (1990). We show that there exists a Markov perfect equilibrium (MPE) in mixed strategy in the game. Furthermore, we give a simple condition for the existence of a unique pure strategy MPE, which admits explicit recursive expressions. We obtain results when customers are myopic as a special case. The explicit characterization of equilibrium behavior allows us to conduct an extensive numerical study to demonstrate our results and gain insights into the problem. Extending known results in the monopolistic setting, we show that dynamic pricing is often undesirable for both firms when customers are strategic, even though it improves profits when customers are myopic. Our results emphasize the role of product quality and the value of price commitment. While both firms tend to suffer profit loss from strategic customer behavior, the low-quality firm often experiences profit loss that is orders of magnitude higher than that of the high-quality firm. The detrimental effects of dynamic pricing are almost fully realized, even when each firm only reduces price once. Furthermore, we study a version of the model where one firm unilaterally commits to static pricing and show that such commitment can be very valuable for the committing firm. Interestingly, when the low-quality firm commits to static pricing, the equilibrium profits are often higher for both firms.

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1. Introduction

Customers often behave strategically in response to firms’ dynamic pricing strategy; that is, they take into account both current and future prices and time their purchases in order to obtain higher surpluses. We have witnessed a growing body of research on such strategic customer behavior and its impact on firms’ pricing and capacity decisions. Much, though not all of the existing literature has primarily focused on monopoly markets. However, there are few products for which the market can be treated as a monopoly. Since customers differ in preferences and tastes, firms often provide differentiated products to compete for profitable niches in heterogeneous markets.

When facing dynamic prices for differentiated products, strategic customers decide not only
which products to purchase, but also when to purchase them. Ideally, a firm should take into consideration both the intra-temporal demand competition and inter-temporal demand substitution. It is therefore desirable to endogenize both customer choice behavior and purchase-timing behavior, capturing their effects on competing firms’ pricing strategies. Yet there is little work that explicitly models the interplay between strategic customer behavior and pricing competition among firms. (Levin et al. (2009) is an exception. We will discuss the differences of our work and theirs in section 2.) Hence, the goal of this paper is to complement the existing literature on strategic customer behavior by considering its impact on firms’ pricing strategy in competitive environments.

To this end, we formulate dynamic pricing competition between two firms as a dynamic game that explicitly captures the effects of strategic customer behavior. In our model, the two competing firms offer vertically differentiated products; that is, one firm offers a high-quality product which is always preferred over a low-quality product provided by the other firm, given equal prices. The firms vary prices over time to maximize their respective expected profit. Customers have heterogeneous valuations on quality, which is the private information of each customer. We assume customers have rational expectations of firms’ future prices, and hence in equilibrium, customers correctly predict the prices charged by the firms. Customers weigh the expected payoffs of purchasing from different firms at different times, and decide when and where to purchase in order to maximize their individual surpluses.

We adopt the classical vertical differentiation model (Tirole 1988), which allows us to explicitly model the intra-temporal (which product to buy) and inter-temporal (when to buy) choice behavior of customers in a dynamic game framework. We show that there exists a Markov perfect equilibrium (MPE) in mixed strategy in our game. Furthermore, we give a simple condition for the existence of a unique pure strategy MPE, which admits explicit recursive expressions. We obtain results for the case when customers are myopic as a special case. We note that, in general, computing MPE in dynamic games is non-trivial. Hence, relatively simple characterization allows us to conduct extensive numerical experiments in order to gain insights into the problem.

Our model can be viewed as a competitive version of the classical model of Besanko and Winston (1990), which considers the impact of strategic customer behavior on dynamic pricing strategies in a monopolistic setting. Similar to the monopolistic case considered in Besanko and Winston (1990), strategic customer behavior tends to reduce the profits of both firms, compared with the case of myopic customers. Our vertical differentiation duopoly model also generates novel insights on how a firm’s product quality affects profit loss resulting from strategic customer behavior. We show that the impact of strategic customer behavior is asymmetric for quality-differentiated firms. The low-quality firm suffers substantially more than the high-quality firm, implying that the disadvantage
of providing low-quality products in vertical differentiation competition is further exacerbated by strategic purchase behavior.

A related question is how equilibrium profits for both firms depend on the number of pricing opportunities. When customers are myopic, it can be shown that the equilibrium profits are increasing in the number of pricing opportunities. However, faced with strategic customers who have incentives to postpone purchases for lower prices, firms are generally worse off as the number of pricing opportunities increases. This result has been discovered in Besanko and Winston (1990) for monopoly markets. More importantly, we find that the detrimental effect of dynamic pricing is almost fully realized, even when each firm changes price only once, especially when customers become more strategic, or the quality levels of the two firms are close.

Since dynamic pricing may hurt firms’ profit, it is natural to consider the commitment to static pricing, where a single price is charged throughout the selling horizon. To address this issue, we model the case in which one or both firms are able to make credible commitments to static pricing and evaluate the value of such commitments in the presence of strategic customers. In general, a strategy that commits to static pricing is not subgame perfect, because a firm always has an incentive to deviate, given the residual demand curve. Nevertheless, the commitment can be enforced through certain commitment devices, such as supply chain contracts (Su and Zhang 2008) and best price provisions (Butz 1990). In unilateral commitment to static pricing, we observe strikingly different outcomes when a high- (or low-) quality firm commits to static pricing. Specifically, when the high-quality firm adopts static pricing, its profit is generally improved to a moderate degree; meanwhile, the low-quality firm suffers significant profit loss on average by dynamically adjusting prices over time. However, when the low-quality firm commits to static pricing unilaterally, both firms can do better!

What if one firm ignores strategic customer behavior, while the other correctly recognizes it? First, it is interesting to observe that the firm who ignores strategic customer behavior suffers profit loss, while its competitor gains. An implication is that firms may not be willing to share information on strategic customer behavior. Second, the cost of ignoring strategic customer behavior is asymmetric. The detrimental effect is much severer for a low-quality firm when it incorrectly assumes myopic purchase behavior of customers. Our numerical studies indicate that the profit loss on average is 0.33% for the high-quality firm, compared with the average profit loss of 22.53% for the low-quality firm. Nevertheless, the profit gains on average are of comparable magnitude for both firms, regardless of who makes correct (or wrong) assumptions of strategic behavior.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 introduces and analyzes the multi-period dynamic pricing competition model. Section 4 considers
cases when one firm unilaterally commits to static pricing, while the other firm can change prices dynamically. Section 5 reports numerical results and managerial insights from our model and analysis. Section 6 summarizes and points out some future research directions. All proofs are in the Appendix.

2. Literature Review

Our work is involved with several key components, including strategic customer behavior, vertical product differentiation, dynamic pricing competition, and price commitment. We review the relevant literature in terms of a combination of those components, and we position our work relative to the literature.

One key component we address in this paper is the aspect of strategic customer behavior, which has received considerable attention in the recent operations management literature. We refer readers to Shen and Su (2007) for an in-depth review of the relevant literature in revenue management. Here we focus on papers dealing with dynamic pricing strategies in the presence of strategic customers. Aviv and Pazgal (2008) consider a problem of selling a fixed amount of seasonal products to customers whose valuations decline over time. They study both inventory-contingent markdown policy and pre-announced fixed-discount policy. Su (2007) looks at a heterogeneous market on two dimensions, including different valuations on the product and different degrees of patience to wait. He shows that different market compositions can result in a markdown or markup pricing policy. Cachon and Swinney (2009) study how a retailer’s stocking quantity and markdown pricing decisions are affected by a portion of customers who strategically wait for sales. They demonstrate the value of a quick response strategy on mitigating the negative consequences of strategic purchase behavior. Lai et al. (2008) investigate the role of posterior price matching policy on a firm’s profitability when faced with strategic customers. Several other papers address strategic customers in a setting of exogenously given prices. For example, Liu and van Ryzin (2008) propose a capacity rationing model to optimally influence the strategic purchase behavior of customers. Yin et al. (2009) show that different in-store display formats (display all or display one) can influence customers’ expectations of availability; thus, their purchase decisions. A few papers, including Ovchinnikov and Milner (2009), Gallego et al. (2008), Liu and van Ryzin (2010), model customer learning about inventory decisions in repeated markdown environments.

However, almost all of the aforementioned papers consider models where strategic customers interact with a single seller. Indeed, there are very few papers that look at competitive setups with more than one seller facing strategic customers. One exception we are aware of is Levin et al. (2009). They formulate the oligopolistic dynamic pricing competition as a stochastic dynamic
game where multiple capacitated firms sell to customers whose behavior can be characterized by a particular stochastic choice model. They characterize the existence of a mixed strategy MPE under very general technical assumptions. They also establish the existence of a pure strategy MPE for a special case of their model. Their extensive numerical study generates many interesting insights into the problem. Our work complements theirs along several dimensions. Aside from the competitive aspect, our model is closer in spirit to Besanko and Winston (1990). While the customer behavior model in Levin et al. (2009) can be best thought of as an example of horizontal competition, we follow the classical vertical differentiation model (Tirole 1988). The relatively simple setup allows us to formulate the problem and characterize the pure strategy MPE, which can be expressed in recursive expressions. The use of vertical differentiation model enables us to focus on the effect of product quality differentiation and leads to interesting observations along this dimension. Several of our findings echo those in Levin et al. (2009), despite the significant differences in the model setups, lending support to the robustness of their results. Furthermore, we study the case where one firm unilaterally commits to static pricing, while the other firm continues to dynamically adjust prices over time, shedding light on the value of price commitment. We also complement our study with extensive numerical experiments to illustrate our results and to gain further insights.

The literature on strategic customer behavior is closely related to the economics literature that considers inter-temporal price discrimination. In his seminal work, Coase (1972) shows that, for durable goods, a monopolist has to price at marginal cost due to the fact that customers strategically wait for price reductions over time. Stokey (1979) studies the pricing behavior of a monopolist selling a new product under a perfect information setting. Lazear (1986) addresses how the inter-temporal pricing strategy is related to various assumptions on product characteristics and underlying market conditions. Our model setup closely follows Besanko and Winston (1990), who show that there exist subgame perfect equilibrium (SPE) prices that decline over time when customers discount their future utilities. In equilibrium, high-valuation customers purchase early at higher prices, while low-valuation customers delay purchases to obtain lower prices. They show that the SPE price is less than the single-period profit maximization price, and the firm is worse off as the number of periods increases. Our work extends theirs to a duopoly model. Our results reinforce many of their findings in the monopolistic setting while also yield interesting insights into the effect of product quality and the value of price commitment.

Product differentiation is an important topic in marketing and economics. Product differentiation can be divided into two types: horizontal and vertical differentiation. Both types of product differentiation are widely observed in practice and are extensively studied in the literature. An
excellent treatment of product differentiation using discrete choice theory is offered by Anderson et al. (1992). The vertical differentiation model we adopted follows Tirole (1988). The model is originally introduced by Mussa and Rosen (1978) to study quality and pricing competition. Time is usually not directly modeled in vertical differentiation models, but can be incorporated as another dimension in order to model inter-temporal behavior. One example is the work by Parlaktürk (2009), which considers a single firm selling vertically differentiated products to strategic customers in two periods. He examines the value of product variety on a firm’s profitability in the presence of strategic customer behavior. Our inter-temporal behavior model is very similar to the one in Parlaktürk (2009), but we consider a multi-period duopoly market instead of a monopoly market in two periods.

Many authors consider competitive dynamic pricing under customer choice behavior. For the theory of oligopoly pricing via a game-theoretical approach, we refer readers to the book by Vives (1999). Xu and Hopp (2006) study a dynamic pricing problem in which retailers determine order quantities prior to sales, and customer arrivals follow a geometric Brownian motion. They establish a weak perfect Bayesian equilibrium for the price and inventory replenishment game under oligopolistic competition. Lin and Sibdari (2009) consider a finite-horizon discrete-time dynamic pricing competition model where customer demand follows a discrete-choice model. They establish the existence of Nash equilibria in the game. Gallego and Hu (2006) consider a similar problem of multiple capacity providers selling differentiated perishable products over a finite time horizon. They incorporate pricing and capacity allocation decisions into a multi-player and non-cooperative stochastic game and establish the existence of open-loop and closed-loop Nash equilibria. Using a different approach of robust optimization, Perakis and Sood (2006) study a multi-period pricing problem for a single perishable product with a fixed inventory level in an oligopolistic market. More recently, Martínez-de-Albéniz and Talluri (2010) consider a finite-horizon competitive dynamic pricing model with fixed starting inventory and homogeneous products. They establish the existence of a unique subgame perfect equilibrium and study its properties. In all of those competitive dynamic pricing models, customers decide where (which product) to buy based on prices and inventory levels available at the time of purchase, and therefore behave myopically. In our modeling framework, the case when all customers are myopic can be treated as a special case. We show that dynamic pricing improves equilibrium profits when customers are myopic, confirming the existing results in the aforementioned papers. However, when customers act strategically, dynamically changing prices can hurt a firm’s profits. Therefore, strategic customer behavior changes the equilibrium outcome qualitatively, emphasizing the importance of understanding such behavior in competitive markets.
Competitive dynamic pricing models build on the huge body of literature on dynamic pricing for multiple products without competition; see, e.g., Gallego and van Ryzin (1997), Dong et al. (2007), and Akçay et al. (2009). The work of Akçay et al. (2009) is particularly relevant in that it also considers vertical product differentiation. They demonstrate that optimal pricing policies are different under horizontal and vertical product differentiation. For two excellent reviews of the relevant literature, see Bitran and Caldentey (2003) and Elmaghraby and Keskinocak (2003).

Commitments are widely studied in the economics and operations literature. The one most relevant to our paper is price commitment. It has been recognized that when customers are strategic, commitment to price or quantity can improve a firm’s revenue. Su and Zhang (2008) demonstrate this point in a supply chain facing strategic customers. They show how commitments can be credibly made through various supply chain contracts. Another way to implement a price commitment is via a best price provision or price guarantee where customers receive a refund whenever a price drops below the amount paid; see Butz (1990) and Levin et al. (2007). Our results on price commitment is consistent with the literature. However, we discover that the effect of price commitment is different for quality-differentiated firms. In general, the price commitment of the high-quality firm improves its own profit, but hurts the low-quality firm on average. However, the price commitment of the low-quality firm can improve the profits of both firms.

3. The Model

Consider a market with two firms, firm H and firm L, with each offering one product. Firm $i$ offers product $i$, $i = H, L$. Product $i$ is characterized by a quality index $q_i$. We assume $q_H > q_L$ throughout; therefore, product H has higher quality. Without loss of generality, we normalize $(q_H, q_L)$ to $(1, \beta)$ with $0 < \beta < 1$. The selling season for the products is divided into $T$ consecutive periods. Time is counted forward, so the first period is period 1, and the last period is period $T$. The products can be sold at different prices in different time periods. The prices offered in period $t$ is denoted by $p_t = (p_{t,H}, p_{t,L})$, $t = 1, \ldots, T$. The per-period discount factor for each firm is $\alpha$ ($0 \leq \alpha \leq 1$). The firms determine prices simultaneously at the beginning of each period to maximize their respective profits collected over $T$ periods.

All customers arrive at the beginning of the selling season prior to period 1. The total number of customers is normalized to 1. Customers have heterogenous valuations (tastes) on quality, denoted by $\theta$, which is private information for each customer (Tirole 1988). We assume that $\theta$ follows a Uniform distribution on $[0, 1]$, which is common knowledge for the firms and customers. If a customer with valuation $\theta$ purchases product $i$ at price $p_{t,i}$ in period $t$, she earns a surplus of $\theta q_i - p_{t,i}$, $i = H, L$; she can also choose not to purchase and earn zero surplus. Customers have a
per-period discount factor $\gamma$ ($0 \leq \gamma \leq 1$). We can interpret $\gamma$ as the level of strategic behavior; a higher $\gamma$ means that customers are more strategic. We assume customers are inter-temporal utility maximizers and purchase at most once during the entire selling season.

We assume linear cost where the per-unit cost of H and L is $c$ and $\beta c$, respectively. To ensure nonnegative profits for the firms, we assume $c < 1$. Our cost assumption can be viewed as a special case of those made in the literature. In the context of sequential quality-price competition for vertically differentiated products, Motta (1993) uses two different assumptions on cost. In the first one, there is a fixed cost, but no variable cost for each quality level, while in the second one, there is a variable cost, but no fixed cost. Since we assume quality levels are exogenously given in our model, fixed cost can be reasonably assumed away when deriving firms’ pricing strategy in equilibrium. For the second case, our linear cost structure is a special case of the often-assumed convex variable cost in the literature ever since Mussa and Rosen (1978). We note that even though our cost assumption is somewhat specific, it does not prevent us from deriving key insights from our model. It would be clear later on from our analysis that a more general cost assumption, although desirable, would render the problem analytically intractable, and therefore, is not warranted. For this reason, we leave the extension to a more general cost structure to future work.

Before proceeding, we note that most of our assumptions follow Besanko and Winston (1990). The main difference is that they consider a single firm, while we consider the competition between two firms offering differentiated products. In this sense, our model can be viewed as a generalization of their model to a competitive setup.

We formulate the model as a finite horizon dynamic game where the state variable is the number of remaining customers in the market. Since customers discount future utilities, if a customer with valuation $\theta'$ purchases in an earlier period, then all customers with valuations greater than $\theta'$ must also purchase in earlier periods. Therefore the remaining customers at each period $t$ can be characterized by an interval $[0, \theta_t]$, where $\theta_t$ is the marginal valuation at which a customer is indifferent between purchasing in period $t-1$ and period $t$. (For convenience, we choose to use a closed interval $[0, \theta_t]$ instead of the half-open interval $[0, \theta_t)$. Since customers with valuation $\theta_t$ have zero measure, this does not affect our analysis or results.) The value $\theta_t$ can be used as the state in period $t$. It is straightforward to show that when all remaining customers have valuations less than $c$, i.e., $\theta_t < c$, there will be no sales for each firm. To avoid such a trivial case, we assume $\theta_t \geq c$. It can be verified that $\theta_t$ will never fall below $c$ on an equilibrium path; therefore, this assumption is without loss of generality. The solution concept we adopt is the Markov perfect equilibrium (MPE), which is a profile of Markov strategies that is subgame perfect for each player; see Fudenberg and Tirole (1991).
3.1 Analysis for the Last Period

We first analyze the game in the last period. Suppose the state is \( \theta_T \in [0, 1] \). Given \( \theta_T \) and a price pair \( p_T = (p_{T,H}, p_{T,L}) \), a customer with valuation \( \theta \leq \theta_T \) purchases product H if it leads to positive surplus that is higher than purchasing product L; i.e., \( \theta - p_{T,H} \geq (\beta \theta - p_{T,L})^+ \), where the notation \((x)^+\) denotes the nonnegative part of \(x\). Similarly, a customer with valuation \( \theta \leq \theta_T \) purchases product L if \( \beta \theta - p_{T,L} > (\theta - p_{T,H})^+ \). Considering all possible consumer purchase decisions for each price pair, we can write the profit for firm H as

\[
\begin{align*}
    r_{T,H}(\theta_T, p_T) &= \begin{cases} 
        0, & \text{if } p_{T,H} \geq p_{T,L} + (1 - \beta)\theta_T, \\
        (p_{T,H} - c)\left(\theta_T - \frac{p_{T,H} - p_{T,L}}{1 - \beta}\right), & \text{if } \frac{p_{T,L}}{\beta} \leq p_{T,H} < p_{T,L} + (1 - \beta)\theta_T, \\
        (p_{T,H} - c)(\theta_T - p_{T,H}), & \text{if } p_{T,H} < \frac{p_{T,L}}{\beta}.
    \end{cases}
\end{align*}
\]

The corresponding profit for firm L is given by

\[
\begin{align*}
    r_{T,L}(\theta_T, p_T) &= \begin{cases} 
        (p_{T,L} - \beta c)\left(\theta_T - \frac{p_{T,L}}{\beta}\right), & \text{if } p_{T,L} \geq p_{T,H} + (1 - \beta)\theta_T, \\
        (p_{T,L} - \beta c)\left(\frac{p_{T,H}}{1 - \beta} - \frac{p_{T,L}}{\beta}\right), & \text{if } \frac{p_{T,L}}{\beta} \leq p_{T,H} < p_{T,L} + (1 - \beta)\theta_T, \\
        0, & \text{if } p_{T,H} < \frac{p_{T,L}}{\beta}.
    \end{cases}
\end{align*}
\]

Proposition 1 shows that there exists a unique Nash equilibrium in the last period. To simplify presentation, we define

\[
\begin{align*}
    A_{T,H} &= \frac{2(1 - \beta)}{4 - \beta}, \quad A_{T,L} = \frac{\beta(1 - \beta)}{4 - \beta}, \quad B_{T,H} = \frac{4(1 - \beta)}{(4 - \beta)^2}, \quad B_{T,L} = \frac{\beta(1 - \beta)}{(4 - \beta)^2}.
\end{align*}
\] (1)

These notations will be used throughout the paper.

**Proposition 1.** Suppose the remaining customers in the last period have valuations in the range \([0, \theta_T]\) where \(\theta_T \geq c\). The unique Nash equilibrium in the last period is given by the price pair

\[
\begin{align*}
    p_{T,H}^*(\theta_T) &= A_{T,H}(\theta_T - c) + c, \quad p_{T,L}^*(\theta_T) = A_{T,L}(\theta_T - c) + \beta c
\end{align*}
\] (2)

with corresponding equilibrium profits

\[
\begin{align*}
    r_{T,H}^*(\theta_T) &= B_{T,H}(\theta_T - c)^2, \quad r_{T,L}^*(\theta_T) = B_{T,L}(\theta_T - c)^2.
\end{align*}
\] (3)

3.2 Analysis for the Second-to-Last Period

In this section, we analyze the game in the second-to-last period \(T - 1\) with state \(\theta_{T-1}\). A two-period game is substantially more complex because we need to consider inter-temporal customer choice between two products over two periods, where each customer decides on which product to purchase and when to purchase in order to maximize her surplus. A customer can also decide not to purchase, in which case she receives zero surplus.
Let $\theta^*_T$ be the valuation of a marginal customer who is indifferent between purchasing in period $T$ and period $T-1$. Note that $\theta^*_T$ depends on the prices in the period $T-1$. Given $\theta^*_T$, the equilibrium prices in period $T$ are given by Proposition 1, which are denoted by $p^*_T(\theta^*_T) = (p^*_{T,H}(\theta^*_T), p^*_{T,L}(\theta^*_T))$.

Suppose the price pair in period $T-1$ is $p_{T-1} = (p_{T-1,H}, p_{T-1,L})$. A customer with valuation $\theta$ buys product H in period $T-1$ if it leads to the highest surplus among all purchase options; formally,

$$\theta - p_{T-1,H} \geq \beta \theta - p_{T-1,L},$$
$$\theta - p_{T-1,H} \geq \gamma(\theta - p^*_{T,H}(\theta^*_T)),$$
$$\theta - p_{T-1,H} \geq \gamma(\beta \theta - p^*_{T,L}(\theta^*_T)),$$
$$\theta - p_{T-1,H} \geq 0.$$  

The right hand sides of the above inequalities are the surpluses of purchasing product L in period $T-1$, purchasing product H in period $T$, purchasing product L in period $T$, and no-purchase, respectively. Similarly, a customer with valuation $\theta$ buys product L in period $T-1$ if and only if

$$\beta \theta - p_{T-1,L} > \theta - p_{T-1,H},$$
$$\beta \theta - p_{T-1,L} > \gamma(\theta - p^*_{T,H}(\theta^*_T)),$$
$$\beta \theta - p_{T-1,L} > \gamma(\beta \theta - p^*_{T,L}(\theta^*_T)),$$
$$\beta \theta - p_{T-1,L} \geq 0. \quad (5)$$

Depending on the relationship between $p_{T-1,H}$ and $p_{T-1,L}$, and values of $\beta$ and $\gamma$, firm L may or may not incur demand in period $T-1$. When firm L incurs positive demand, the marginal valuation $\theta^*_T$ is determined by comparing the surpluses of purchasing L in period $T-1$ and purchasing H in period $T$, satisfying $\beta \theta^*_T - p_{T-1,L} = \gamma(\theta^*_T - p^*_{T,H}(\theta^*_T))$. On the other hand, if firm L does not incur positive demand in period $T-1$, $\theta^*_T$ is determined by comparing the surpluses of purchasing H in period $T-1$ and purchasing H in period $T$, satisfying $\theta^*_T - p_{T-1,H} = \gamma(\theta^*_T - p^*_{T,H}(\theta^*_T))$. In the following, we consider the two cases separately in order to write down the payoff functions of both firms.

**Case 1: L has positive demand in period $T-1$.** This requires that (i) there is a nonempty set of $\theta$ satisfying (4) and (5); (ii) the period-T marginal valuation $\theta^*_T$ is determined by $\beta \theta^*_T - p_{T-1,L} = \gamma(\theta^*_T - p^*_{T,H}(\theta^*_T))$. We can easily show that condition (i) holds if and only if $p_{T-1,L} < \beta p_{T-1,H}$. Furthermore, by substituting the equilibrium price in the last period $p^*_{T,H}$, we have

$$\beta \theta^*_T - p_{T-1,L} = \gamma(\theta^*_T - p^*_{T,H}(\theta^*_T)) \iff \beta \theta^*_T - p_{T-1,L} = \gamma(\theta^*_T - A_{T,H}(\theta^*_T) - c) - c \iff [\beta - \gamma(1 - A_{T,H})] \theta^*_T = p_{T-1,L} - \gamma(1 - A_{T,H})c.$$

Hence, when $\beta - \gamma(1 - A_{T,H}) > 0$, there exists a feasible $\theta^*_T$ determined by $\theta^*_T = \frac{p_{T-1,L} - \gamma(1 - A_{T,H})c}{\beta - \gamma(1 - A_{T,H})}$. 


Case 2: L has 0 demand in period $T - 1$. Similar to the analysis in Case 1, we conclude that when $\beta - \gamma(1 - A_{T,H}) \leq 0$ or $p_{T-1,L} \geq \beta p_{T-1,H}$, firm L does not incur sales in period $T - 1$. The period-T marginal valuation $\theta^*_T$ is then determined by the indifference point between purchasing H in periods $T - 1$ and $T$; that is,

$$
\theta^*_T - p_{T-1,H} = \gamma(\theta^*_T - p^*_{T,H}(\theta^*_T)) \Leftrightarrow \theta^*_T = \frac{p_{T-1,H} - \gamma(1 - A_{T,H})c}{1 - \gamma(1 - A_{T,H})}.
$$

Based on the analysis above, the payoff functions of the two players, $r_{T-1,i}(\theta_{T-1}, p_{T-1})$, $i = H, L$, can be written as

$$
\begin{align*}
    r_{T-1,H}(\theta_{T-1}, p_{T-1}) &= \begin{cases} 
        (p_{T-1,H} - c) \left[ \theta_{T-1} - \frac{p_{T-1,H} - p_{T-1,L}}{1-\beta} \right]^+ + \alpha r_{T,H} \left( \frac{p_{T-1,L} - \gamma(1 - A_{T,H})c}{\beta - \gamma(1 - A_{T,H})} \right), & \text{if } p_{T-1,L} < \beta p_{T-1,H} \text{ and } \beta - \gamma(1 - A_{T,H}) > 0, \\
        (p_{T-1,H} - c) \left[ \theta_{T-1} - \frac{p_{T-1,H} - \gamma(1 - A_{T,H})c}{1 - \gamma(1 - A_{T,H})} \right]^+ + \alpha r_{T,H} \left( \frac{p_{T-1,L} - \gamma(1 - A_{T,H})c}{1 - \gamma(1 - A_{T,H})} \right), & \text{otherwise}; \\
    \end{cases}
    \\
    r_{T-1,L}(\theta_{T-1}, p_{T-1}) &= \begin{cases} 
        (p_{T-1,L} - \beta c) \left[ \frac{p_{T-1,H} - p_{T-1,L}}{1-\beta} - \frac{p_{T-1,L} - \gamma(1 - A_{T,H})c}{\beta - \gamma(1 - A_{T,H})} \right]^+ + \alpha r_{T,L} \left( \frac{p_{T-1,L} - \gamma(1 - A_{T,H})c}{\beta - \gamma(1 - A_{T,H})} \right), & \text{if } p_{T-1,L} < \beta p_{T-1,H} \text{ and } \beta - \gamma(1 - A_{T,H}) > 0, \\
        \alpha r_{T,L} \left( \frac{p_{T-1,L} - \gamma(1 - A_{T,H})c}{1 - \gamma(1 - A_{T,H})} \right), & \text{otherwise}.
    \end{cases}
\end{align*}
$$

The following proposition characterizes the equilibrium in period $T - 1$.

**Proposition 2.** The Nash equilibrium solution in period $T - 1$ can be characterized as follows:

(i) When $\gamma \leq \frac{\beta}{1+\beta} \left( 4 - \beta - \frac{2\gamma(1 - \beta)}{4 - \beta} \right)$, in equilibrium both firms incur sales in period $T - 1$, and the equilibrium prices are given by
\[ p^*_{T-1,H}(\theta_{T-1}) = \frac{p^*_{T-1,L}(\theta_{T-1}) + (1 - \beta)\theta_{T-1} + c}{2(1 - \beta)[\beta - \gamma(1 - A_{T,H})] + 4(1 - \beta)[\beta - \gamma(1 - A_{T,H})] - 4\alpha(1 - \beta)B_{T,L} + \beta c}, \]

\[ p^*_{T-1,L}(\theta_{T-1}) = \frac{2\theta_{T-1} - c}{3[\beta - \gamma(1 - A_{T,H})] + 4(1 - \beta)[\beta - \gamma(1 - A_{T,H})] - 4\alpha(1 - \beta)B_{T,L}}. \]

(ii) When \( \gamma \geq \frac{\beta(4 - \beta)}{2 + \beta} \), in equilibrium only product \( H \) incurs sales in period \( T - 1 \) and the equilibrium prices are given by

\[ p^*_{T-1,H}(\theta_{T-1}) = \frac{[1 - \gamma(1 - A_{T,H})] + c}{2[1 - \gamma(1 - A_{T,H}) - \beta B_{T,L}]} + c; \]

\[ p^*_{T-1,L}(\theta_{T-1}) = \text{any value in } [\beta c, \beta \theta_{T-1}]. \]

(iii) When \( \frac{\beta}{1 + \beta} \left(4 - \beta - \frac{2\alpha(1 - \beta)}{4 - \beta}\right) < \gamma < \frac{\beta(4 - \beta)}{2 + \beta} \), there exists a mixed-strategy equilibrium in period \( T - 1 \).

Recall that \( \beta \) is the relative quality level of the two products, and \( \gamma \) measures the strategic level of customers. Proposition 2 demonstrates the critical role that \( \beta \) and \( \gamma \) play in determining the equilibrium outcome. Given the qualities of the two products, for a smaller \( \gamma \) – implying less strategic customers – both firms incur sales in the current period in a pure-strategy equilibrium. When customers become highly strategic, high-quality firm \( H \) could price at a certain value such that all customers prefer to purchase from it in the current period, regardless of its competitor’s prices. However, when the strategic level of customers lies in a middle range, both equilibria may occur, and an equilibrium in mixed strategy exists.

Figure 1 illustrates the equilibrium outcome in period \( T - 1 \) for \( \alpha = 1 \). We note that a mixed-strategy equilibrium exists when \( \beta \) and \( \gamma \) are relatively close. Further, according to Part (iii) in Proposition 2, the region with a mixed-strategy equilibrium shrinks as \( \alpha \) decreases and becomes empty when \( \alpha = 0 \).

### 3.3 Multi-Period Analysis

In this section, we analyze the \( T \)-period game for \( T \geq 2 \). We first establish the existence of a mixed-strategy MPE, which is summarized in Theorem 1. Furthermore, we show that under appropriate assumptions on \( \beta \) and \( \gamma \), there exists a unique pure-strategy MPE.

**Theorem 1.** There exists a mixed-strategy MPE for the \( T \)-period dynamic game.

A mixed-strategy MPE specifies a probability distribution over the strategy space for each player, given the state and the Markov strategy of other players. While the existence result in Theorem 1 is reassuring, it is well-known in the game theory literature that a mixed-strategy can be difficult to interpret and implement. In order to refine the result, we establish the technical conditions under which a pure-strategy MPE exists. This is done in two steps. First, in Proposition 3, we
establish the conditions under which a pure-strategy equilibrium exists in period $t$, given that a pure-strategy MPE exists from period $t+1$ onwards. We then show in Theorem 2 that the technical conditions are satisfied by simply requiring that $\beta \geq \gamma$.

**Proposition 3.** Consider the game at period $t$ with state $\theta_t$. Suppose the Markov equilibrium in period $t+1$ is a pure strategy with

$$p_{t+1,H}^*(\theta_{t+1}) = A_{t+1,H}(\theta_{t+1} - c) + c, \quad r_{t+1,H}^*(\theta_{t+1}) = B_{t+1,H}(\theta_{t+1} - c)^2,$$

$$p_{t+1,L}^*(\theta_{t+1}) = A_{t+1,L}(\theta_{t+1} - c) + \beta c, \quad r_{t+1,L}^*(\theta_{t+1}) = B_{t+1,L}(\theta_{t+1} - c)^2,$$

where $A_{t+1,H}$, $A_{t+1,L}$, $B_{t+1,H}$, and $B_{t+1,L}$ are strictly positive constants, and $\theta_{t+1}$ is the state in period $t+1$. To simplify notations, define

$$X_{t+1} = \beta - \gamma(1 - A_{t+1,H}), \quad \Delta_{t+1} = 3X_{t+1}^2 + 4(1 - \beta)X_{t+1} - 4\alpha(1 - \beta)B_{t+1,L}.$$

We have the following results regarding the equilibrium in period $t$:

(i) If $\beta - \gamma(1 - A_{t+1,H}) - 2\alpha B_{t+1,L} \geq 0$ and $B_{t+1,H} - B_{t+1,L} \leq \frac{1 - \beta}{2\alpha}$, there exists a pure-strategy equilibrium in period $t$ at which both firms have positive demand, and the equilibrium prices and profits can be characterized by

$$p_{t,H}^*(\theta_t) = A_{t,H}(\theta_t - c) + c, \quad r_{t,H}^*(\theta_t) = B_{t,H}(\theta_t - c)^2,$$

$$p_{t,L}^*(\theta_t) = A_{t,L}(\theta_t - c) + c, \quad r_{t,L}^*(\theta_t) = B_{t,L}(\theta_t - c)^2,$$

where

$$A_{t,H} = \frac{(1 - \beta)(X_{t+1}^2 + \Delta_{t+1})}{2\Delta_{t+1}},$$

$$A_{t,L} = \frac{(1 - \beta)X_{t+1}^2}{\Delta_{t+1}},$$

$$B_{t,H} = \frac{(1 - \beta)([\Delta_{t+1} + X_{t+1}^2] + 4\alpha(1 - \beta)B_{t+1,H}X_{t+1}^2)}{4\Delta_{t+1}^2},$$

$$B_{t,L} = \frac{(1 - \beta)X_{t+1}^2(\Delta_{t+1} - X_{t+1}^2 - 2(1 - \beta)X_{t+1} + 2\alpha(1 - \beta)B_{t+1,L})}{2\Delta_{t+1}^2}.$$

(ii) If $\beta - \gamma(1 - A_{t+1,H}) \leq 0$, only firm $H$ has positive demand in period $t$, and the pure-strategy equilibrium in period $t$ can be characterized by

$$p_{t,H}^*(\theta_t) = A_{t,H}(\theta_t - c) + c, \quad r_{t,H}^*(\theta_t) = \frac{A_{t,H}}{2}(\theta_t - c)^2,$$

$$p_{t,L}^*(\theta_t) = \text{any value in } [\beta c, \beta \theta_0], \quad r_{t,L}^*(\theta_t) = B_{t,L}(\theta_t - c)^2,$$

where

$$A_{t,H} = \frac{(1 - \beta + X_{t+1})^2}{2[1 - \beta + X_{t+1} - \alpha B_{t+1,H}]},$$

$$B_{t,L} = \frac{\alpha B_{t+1,L}[1 - \beta + X_{t+1}]^2}{4[1 - \beta + X_{t+1} - \alpha B_{t+1,H}]^2}.$$
(iii) If $0 < \beta - \gamma(1 - A_{t+1,H}) < 2\alpha B_{t+1,L}$, or $\beta - \gamma(1 - A_{t+1,H}) > 0$ and $B_{t+1,H} - B_{t+1,L} > \frac{1-\alpha}{2}$, there exists a mixed-strategy equilibrium in period $t$.

Proposition 3 fully characterizes equilibria in period $t$ over the entire strategy space, given that a pure-strategy MPE exists in period $t+1$. It reveals how the system parameters, including the relative quality level $\beta$ and strategic level of customers $\gamma$, affect the equilibrium outcome. In general, there are two equilibria in which both firms incur sales, or only the high-quality firm incurs sales. Firms may randomize over these two equilibria and play a mixed strategy. However, we are able to show that a unique pure-strategy MPE involving positive demand from both firms in each period exists for a $T$-period game under a simple condition, which is established in the following theorem.

**Theorem 2.** Suppose $\gamma \leq \beta$. There exists a unique pure-strategy MPE in the dynamic game. Let

$$X_t = \beta - \gamma(1 - A_{t,H}), \quad \Delta_t = 3X_t^2 + 4(1 - \beta)X_t - 4\alpha(1 - \beta)B_{t,L}, \quad \forall t = 1, \ldots, T.$$

The equilibrium prices and associated profits can be characterized by

$$p_{t,H}^*(\theta_t) = A_{t,H}(\theta_t - c) + c, \quad r_{t,H}^*(\theta_t) = B_{t,H}(\theta_t - c)^2,$$

$$p_{t,L}^*(\theta_t) = A_{t,L}(\theta_t - c) + \beta c, \quad r_{t,L}^*(\theta_t) = B_{t,L}(\theta_t - c)^2,$$

where $A_{T,H}, A_{T,L}, B_{T,H}$ and $B_{T,L}$ are given in (1); and $A_{t,H}, A_{t,L}, B_{t,H}$ and $B_{t,L}$ are defined in (6)–(9) for $t = 1, \ldots, T - 1$.

Theorem 2 shows that when $\gamma \leq \beta$, a unique pure-strategy MPE exists and can be characterized by explicit recursive equations. These recursive expressions enable us to solve the equilibrium easily. The sequence of price and profit coefficients, $\{A_{t,i}\}_{t=1}^{T-1}$, $\{B_{t,i}\}_{t=1}^{T-1}$, $i = H, L$, are calculated backwards according to (6)–(9). Note that $\theta_1 = 1$, the sequence of states, $\{\theta_t\}_{t=2}^T$, are then determined forward, based on the equilibrium prices in the last period $(p_{T-1,H}, p_{T-1,L})$. Hence, we are able to efficiently conduct extensive numerical experiments to generate a number of managerial insights from the model, which are reported later in Section 5.

The condition $\gamma \leq \beta$ is a natural one. Recall $\gamma$ is the one-period discount factor, and $\beta$ is the quality ratio of the two products. The relationship between $\gamma$ and $\beta$ roughly captures the relative attractiveness of buying the low-quality product now versus buying the high-quality product in the next period. The condition $\gamma \leq \beta$ implies that customers prefer purchasing low-quality product L in the current period over buying high-quality product H in the next period when these two options are equally priced. When this condition is not satisfied, product L is not competitive enough, as even buying H in the next period is more appealing than buying L in the current period. In this case, an equilibrium may involve only H incurring sales in a given period (see Proposition 3).
In the MPE, we would expect that equilibrium prices decrease over time for each firm. This is because all of the customers who purchased in period \( t + 1 \) would have purchased in period \( t \) if they anticipate an increase in price in period \( t + 1 \). The following proposition provides a formal statement of this result.

**Proposition 4.** Suppose \( \gamma \leq \beta \). In the pure-strategy MPE, prices decrease over time for each firm; that is, for \( 1 \leq t \leq T - 1 \),
\[
p^*_t;i \geq p^*_{t+1;i}, \quad i = H, L.
\]

Proposition 4 can be viewed as a generalization of Proposition 2 in Besanko and Winston (1990), where they establish the same result in monopoly markets. We conclude that price skimming arises under competition, as well.

**Proposition 5.** Suppose \( \gamma \leq \beta \). As \( \beta \) increases to 1, \( p^*_t;H \) approaches \( c \) and \( p^*_t;L \) approaches \( \beta c \), and both firms earn zero profits.

This result is fairly intuitive. When two firms offer almost identical products, perfect competition leads to market prices equal to marginal costs, and hence, zero profits in equilibrium.

### 3.4 Equilibrium Results when Customers are Myopic

When customers are myopic, they ignore future price expectations and make purchase decisions based on current prices only. In our model, myopic customers purchase as long as they earn a positive surplus of buying either product \( H \) or product \( L \), implying \( \gamma = 0 \). Then an immediate consequence of Theorem 2 establishes the existence of a unique pure-strategy MPE in the following:

**Proposition 6.** With myopic customers, the unique pure-strategy MPE can be described as follows:
\[
\tilde{p}^*_t;H(\theta_t) = \tilde{A}_{t;H}(\theta_t - c) + c, \quad \tilde{r}^*_t;H(\theta_t) = \tilde{B}_{t;H}(\theta_t - c)^2, \\
\tilde{p}^*_t;L(\theta_t) = \tilde{A}_{t;L}(\theta_t - c) + \beta c, \quad \tilde{r}^*_t;L(\theta_t) = \tilde{B}_{t;L}(\theta_t - c)^2.
\]

where
\[
\tilde{A}_{T;H} = \frac{2(1 - \beta)}{4 - \beta}, \quad \tilde{A}_{T;L} = \frac{\beta(1 - \beta)}{4 - \beta}, \quad \tilde{B}_{T;H} = \frac{4(1 - \beta)}{(4 - \beta)^2}, \quad \tilde{B}_{T;L} = \frac{\beta(1 - \beta)}{(4 - \beta)^2},
\]
and for \( t = 1, \ldots, T - 1 \),
\[
\tilde{A}_{t;H} = \frac{2(1 - \beta)(\beta - \alpha(1 - \beta)\tilde{B}_{t+1;L})}{4\beta - \beta^2 - 4\alpha(1 - \beta)\tilde{B}_{t+1;L}}, \quad \tilde{A}_{t;L} = \frac{(1 - \beta)\beta^2}{4\beta - \beta^2 - 4\alpha(1 - \beta)\tilde{B}_{t+1;L}},
\]

(10)

(11)
\[ \tilde{B}_{t,H} = \frac{(1 - \beta)(4(\beta - \alpha(1 - \beta)\tilde{B}_{t+1,L}))^2 + \alpha(1 - \beta)\beta^2\tilde{B}_{t+1,H}}{(4\beta - \beta^2 - 4\alpha(1 - \beta)\tilde{B}_{t+1,L})^2}, \]  
(12)

\[ \tilde{B}_{t,L} = \frac{(1 - \beta)\beta^2(\beta - \alpha(1 - \beta)\tilde{B}_{t+1,L})}{(4\beta - \beta^2 - 4\alpha(1 - \beta)\tilde{B}_{t+1,L})^2}. \]  
(13)

A careful examination of the equations (10)-(13) reveals that those coefficients decline monotonically over time, which is established in Proposition 7 below.

**Proposition 7.** For \( T - 1 \geq t \geq 1 \), \( \tilde{A}_{t,i} \geq \tilde{A}_{t+1,i} \), and \( \tilde{B}_{t,i} \geq \tilde{B}_{t+1,i}, \ i = H, L \).

Proposition 7 shows that the equilibrium prices and profits are decreasing in time for both firms. One implication is that when the selling horizon is lengthened, the equilibrium profits for both firms increase. Therefore, firms benefit from more selling opportunities faced with myopic customers. Intuitively, demand is less price elastic with myopic customers; thus, firms are able to exploit customer surplus by price skimming. However, this is, in general, not true when customers are strategic since high-value customers may postpone purchases to wait for lower prices. Hence, firms may suffer from price flexibility due to such inter-temporal customer behavior. We will provide a more detailed discussion in Section 5.

### 4. Unilateral Commitment to Static Pricing

Although dynamic pricing enables the firm to price discriminate among customers, it also provides an incentive for strategic customers to postpone purchases and wait for lower prices. Hence, dynamic pricing may result in profit loss relative to the case of charging a single price. We call the policy that charges a single price over the entire selling horizon a static pricing policy. Given the possible negative consequences of dynamic pricing, firms may have an incentive to commit to a static pricing policy. We note that the commitment to static pricing may not be credible because a firm may have an incentive to deviate by charging a different price, faced with the residual demand. The commitment can be made credible through certain commitment devices, such as supply chain contracts (Su and Zhang 2008) and best price provisions (Butz 1990). In this section, we consider the value of commitment to static pricing (or price commitment, for short). We assume that one firm commits to static pricing, and the other firm can dynamically adjust prices over time, and both firms determine their first-period prices simultaneously. Note that when both firms commit to static pricing, the problem reduces to the last period problem with \( \theta_T = 1 \), which was analyzed in section 3.1.
4.1 Firm H Commits to Static Pricing

We first consider the case when firm H is able to commit to static pricing, while firm L continues to dynamically change prices over time. That is, firm H offers the product at a fixed price $p_H$ during the course of a sales season; firm L charges a price $p_{t,L}$ at period $t$, $t = 1, ..., T$.

Since a customer discounts its utility over time, she will always prefer to purchase in the first period over the later periods if she decides to buy product H. Therefore, firm H has positive demand only in the first period. Let $\theta_1$ be the cutoff value at which customers are indifferent between purchasing product H and product L; $\theta_1$ satisfies

$$\theta_1 - p_H = \beta \theta_1 - p_{1,L}. \hspace{1cm} (14)$$

The payoff function of H is given by

$$r_H(p_H, p_{1,L}) = (p_H - c)(1 - \theta_1). \hspace{1cm} (15)$$

By (14) and (15), we can easily show that, for a given price $p_{1,L}$, the optimal price charged by firm H, $p_H^*$, is equal to $\frac{p_{1,L} + 1 - \beta + c}{2}$ and $\theta_1 = \frac{1}{2} - \frac{p_{1,L} - c}{2(1 - \beta)}$. On the other hand, firm L incurs sales from period 1 through period $T$, and its equilibrium pricing strategy can be determined by the following optimization problem:

$$\max_{\{p_{t,L}^{\theta_1}, \theta_1\}_{t=1}^T} \sum_{t=1}^T \alpha^{t-1}(p_{t,L} - \beta c)(\theta_t - \theta_{t+1}) \hspace{1cm} (16)$$

s.t. \hspace{1cm}

$$\theta_1 = \frac{1}{2} - \frac{p_{1,L} - c}{2(1 - \beta)}, \hspace{1cm} (17)$$

$$\beta \theta_t - p_{t-1,L} = \gamma(\beta \theta_t - p_{t,L}), \hspace{1cm} t = 2, ..., T, \hspace{1cm} (17)$$

$$\beta \theta_{T+1} - p_{T,L} = 0, \hspace{1cm} (18)$$

$$\theta_t \geq \theta_{t+1}, \hspace{1cm} t = 1, ..., T. \hspace{1cm} (19)$$

In the above, constraint (17) defines the marginal valuation $\theta_t$ at which a customer is indifferent between purchasing in period $t$ and period $t - 1$; constraint (18) means that remaining customers with non-negative surpluses purchase in period $T$.

**Proposition 8.** Suppose firm H commits to static pricing, and firm L can dynamically change prices over $T$ discrete periods. A Markov perfect equilibrium exists and can be described as follows:

$$p_H^* = \frac{1}{2}(1 - c)(1 - \beta + C_1) + c, \hspace{1cm} p_{1,L}^* = C_1(1 - c) + \beta c,$$

$$p_{t,L}^*(\theta_t) = C_1(\theta_t - c) + \beta c, \hspace{1cm} t = 2, ..., T, \hspace{1cm} (20)$$

$$r_H^* = \frac{(1 - c)^2(1 - \beta + C_1)^2}{4(1 - \beta)},$$
\[ r^*_L = \frac{1}{4} C_1 (1 - c)^2, \]
\[ r^*_t(\theta_t) = \frac{1}{2} C_t (\theta_t - c)^2, \quad t = 2, \ldots, T, \]

where \( C_t \) is determined by:

\[ C_1 = \frac{1}{2} (1 - \beta)(\beta - \gamma(\beta - C_2))^2 \]
\[ \frac{(\beta - \gamma(\beta - C_2))^2 + 2(1 - \beta)(\beta - \gamma(\beta - C_2)) - \alpha C_2(1 - \beta)}{(\beta - \gamma(\beta - C_2))^2 + 2(1 - \beta)(\beta - \gamma(\beta - C_2)) - \alpha C_2(1 - \beta)}, \quad t = 2, \ldots, T - 1, \]
\[ C_T = \frac{\beta}{2}. \]

From the simple recursive expressions shown in Proposition 8, we can easily calculate the equilibrium prices and associated profits for both firms. A comparison with the case where both firms can change prices dynamically would reveal the value of price commitment. We provide more detailed discussions in Section 5.2.

4.2 Firm L Commits to Static Pricing

We now consider the case in which firm L commits to static pricing, while firm H can change prices dynamically over time. By charging a fixed price, firm L only incurs sales in the first period, according to the same argument as in section 4.1.

A customer may choose to purchase product L in the first period, or to buy product H in period 1, 2, \ldots, \( T \). Such a customer maximizes her utility by comparing the surpluses of all those purchase opportunities. Suppose customers with valuations in \([\hat{\theta}_1, \hat{\theta}_2]\) buy product L, where \( \hat{\theta}_1 \) is the valuation of a marginal customer who is indifferent between purchasing H in period \( t + 1 \) and purchasing L in period 1, and \( \hat{\theta}_2 \) is the valuation of a marginal customer who is indifferent between purchasing H in period \( t \) and purchasing L in period 1. That is,

\[ \beta \hat{\theta}_1 - p_L = \gamma^t (\hat{\theta}_1 - p_{t+1,H}), \tag{22} \]
\[ \beta \hat{\theta}_2 - p_L = \gamma^{t-1} (\hat{\theta}_2 - p_{t,H}). \tag{23} \]

The analysis of the game is potentially very complex because \( t \) in (22) and (23) can take any value in 1, 2, \ldots, \( T \), depending on the values of \( \beta \) and \( \gamma \). However, when \( \beta > \gamma \), we can show that in equilibrium \( t = 1 \), which substantially simplifies the analysis. This results is established in the following lemma.

**Lemma 1.** Suppose firm L commits to static pricing, and firm H can dynamically change prices over \( T \) discrete periods. When \( \beta > \gamma \), in equilibrium, customers purchasing product L in the first period can be characterized by the valuation interval \([\hat{\theta}_1, \hat{\theta}_2]\) where \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \) satisfy (22) and (23) with \( t = 1 \).
This result implies that, in equilibrium, the highest-value segment of customers purchase product H in the first period, and the second-highest-value segment purchases product L in the first period, while all remaining customers purchase product H in periods 2 to T or make no purchase. The result significantly simplifies the analysis of the game and facilitates the characterization of the equilibrium prices and profits shown in the following proposition.

**Proposition 9.** Suppose firm L commits to static pricing, and firm H can dynamically change prices over T discrete periods. When $\beta > \gamma$, a Markov perfect equilibrium exists and can be described as follows:

$$
p^*_L = \frac{(1 - \beta)[(2\beta cT_4 + 2T_5 - c - 1)\gamma + \beta(1 - c)]}{3(1 - \gamma) + (1 - \beta)(1 - 4\gamma T_4)} + c, \quad (24)
$$

$$
p^*_1, H = \frac{(1 - \beta)((T_5 + cT_4 - (2 - c)(1 + T_4 - T_4\beta))\gamma + 2(1 - c))}{3(1 - \gamma) + (1 - \beta)(1 - 4\gamma T_4)} + c, \quad (25)
$$

$$
p^*_2, H = T_4p^*_L + T_5, \quad (26)
$$

$$
p^*_t, H(\theta_t) = D_t(\theta_t - c) + c, \quad t = 3, \ldots, T, \quad (27)
$$

$$
r^*_t, H(\theta_t) = D_t(\theta_t - c)^2, \quad t = 3, \ldots, T, \quad (28)
$$

$$
r^*_L = (p^*_L - \beta c) \left( \frac{p^*_1, H - p^*_L}{1 - \beta} - \frac{p^*_L - \gamma p^*_2, H}{1 - \beta} \right), \quad (29)
$$

$$
r^*_1, H = (p^*_1, H - c) \left( 1 - \frac{p^*_1, H - p^*_L}{1 - \beta} \right) + r^*_2, H, \quad (30)
$$

$$
r^*_2, H = (p^*_2, H - c) \left( \frac{p^*_L - \gamma p^*_2, H}{1 - \beta} - \frac{p^*_2, H + \gamma D_3 c - \gamma c}{1 - \gamma + \gamma D_3} \right) + \frac{\alpha D_3}{2} \left( \frac{p^*_2, H - c}{1 - \gamma + \gamma D_3} \right)^2, \quad (31)
$$

where

$$
D_T = \frac{1}{2}, \quad D_t = \frac{(1 - \gamma + \gamma D_{t+1})^2}{2(1 - \gamma + \gamma D_{t+1}) - \alpha D_{t+1}}, \quad t = 3, \ldots, T - 1, \quad (32)
$$

and

$$
T_1 = \frac{1}{\beta - \gamma}, T_2 = \frac{1}{1 - \gamma + \gamma D_3}, T_3 = \frac{\alpha D_3}{T_1}, T_4 = \frac{T_3}{2\gamma T_1 + 2T_2 - T_3}, T_5 = \frac{cT_3(\gamma - \gamma D_3 - 1) - T_1\gamma c}{2\gamma T_1 + 2T_2 - T_3} + c.
$$

Although the expressions look a bit messy, they can be easily applied to calculate the equilibrium prices and profits in numerical experiments. We will illustrate the results and compare them with the case without price commitment in Section 5.2.

**5. Numerical Study and Managerial Insights**

In this section, we conduct exhaustive numerical experiments and discuss the managerial implications of our models and results. Our experimental design centers around two main contributing factors in the model. The first one is vertical product differentiation, which is summarized by
the parameter $\beta$. The second factor is strategic customer behavior, which is summarized by the parameter $\gamma$. We consider different combinations of $\beta$ and $\gamma$ while fixing $\alpha = 1$ and $c = 0$. We have tried different $\alpha$ and $c$ values and found qualitatively similar results. We consider cases where $\beta \in \{0.01, 0.02, \ldots, 0.99\}$ and $\gamma \in \{0.0, 0.01, \ldots, 0.99\}$ with the requirement that $\gamma \leq \beta$. In total, we consider 5,049 cases with different combinations of $\beta$ and $\gamma$ values. Of course, the equilibrium profits for $H$ and $L$ also depend on the total number of periods $T$. We use $r_T^*(\beta, \gamma)$ to denote the equilibrium profit for firm $i$, given $T$, $\beta$ and $\gamma$; $i = L, H$. In almost all cases, however, we observe that the equilibrium profit $r_T^*(\beta, \gamma)$ converges pretty quickly as $T$ increases. We denote by $r_\gamma^\infty(\beta, \gamma)$ the limiting equilibrium profit for firm $i$ as $T$ goes to infinity for $i = H, L$, given parameters $\beta$ and $\gamma$. We report the limiting equilibrium profits whenever possible to avoid the explicit dependence on $T$.

5.1 Impact of Customer Rationality

We numerically verified that the limiting equilibrium profit $r_\gamma^\infty(\beta, \gamma)$ is decreasing in $\gamma$ for $i = H, L$. See Figure 2 for illustration. In this example, we take $\beta = 0.8$ and plot the percentage loss in profit relative to the case of myopic customers, $\frac{r_\gamma^\infty(0.8, \gamma) - r_\gamma^\infty(0.8, 0)}{r_\gamma^\infty(0.8, 0)} \times 100$, for $0 \leq \gamma \leq 0.8$. We observe that strategic customer behavior tends to reduce the profits of both firms, and the profit loss from strategic customer behavior can be quite significant. This result is not surprising since demand is more price elastic with strategic customers, who now have the option to delay their purchases. Similar observations are made in the earlier paper by Besanko and Winston (1990) in a monopolistic setting.

An equally interesting question is how product quality affects the relative profit loss from strategic customer behavior in a competitive setting. To this end, we fix $\gamma$ and calculate the percentage
loss in profit \( \frac{r_i^{\infty}(\beta, \gamma) - r_i^{\infty}(\beta, 0)}{r_i^{\infty}(\beta, 0)} \times 100 \) for all \( \beta \geq \gamma \). Figure 3 demonstrates the results when \( \gamma = 0.2 \).

We observe that for a fixed \( \gamma \) value, the absolute value of percentage loss in profits is decreasing in \( \beta \). Furthermore, the magnitude of loss is substantially different between firm H and firm L; the profit loss for H is only a fraction of that experienced by L. This implies that the impact of strategic customer behavior is asymmetric for H and L, with L suffering substantially more than H.

5.2 Unilateral Commitment to Static Pricing

Section 4 studies the equilibrium when one firm commits to static pricing, while the other firm continues to use dynamic pricing. Here we numerically study the effects of such commitments. To be consistent with the assumptions made in Section 4, we assume \( \beta \) is strictly larger than \( \gamma \), resulting in 4,950 test cases. We calculate the percentage difference in firm \( i \)'s profit when firm H commits to static pricing according to \( \frac{r_{HC}^{\infty}(\beta, \gamma) - r_i^{\infty}(\beta, \gamma)}{r_i^{\infty}(\beta, 0)} \), \( i = H, L \), where \( r_{HC}^{\infty}(\beta, \gamma) \) refers to the equilibrium profit for firm \( i \) when firm H commits to static pricing, and \( r_i^{\infty}(\beta, \gamma) \) is the equilibrium profit of firm \( i \) when both firms adopt dynamic pricing, and it serves as a benchmark. Similarly, the percentage difference in profit for firm \( i \) when L commits to static pricing is calculated according to \( \frac{r_{LC}^{\infty}(\beta, \gamma) - r_i^{\infty}(\beta, \gamma)}{r_i^{\infty}(\beta, 0)} \), \( i = H, L \), where \( r_{LC}^{\infty}(\beta, \gamma) \) denotes the equilibrium profit for firm \( i \) when firm L commits to static pricing.

Table 1 reports some summary statistics over the 4,950 test cases when either H or L commits to static pricing. When H commits, the percentage difference in profit has an average of 1.89% and ranges between -16.49% and 8.26% for H. The percentage difference in profit has an average of -13.19% and ranges between -572.18% and 15.44% for L. The statistics when L commits are substantially different. The percentage difference in profit has an average of 19.69% and ranges between -7.15% and 39.03% for H, while for L the percentage difference in profit has an average
<table>
<thead>
<tr>
<th>Firm</th>
<th>H Commits</th>
<th>L Commits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
<td>L</td>
</tr>
<tr>
<td>Maximum</td>
<td>8.26%</td>
<td>15.44%</td>
</tr>
<tr>
<td>Minimum</td>
<td>-16.49%</td>
<td>-572.18%</td>
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<tr>
<td>Average</td>
<td>1.89%</td>
<td>-13.19%</td>
</tr>
<tr>
<td>Third Quartile</td>
<td>4.05%</td>
<td>6.50%</td>
</tr>
<tr>
<td>Median</td>
<td>3.08%</td>
<td>2.83%</td>
</tr>
<tr>
<td>First Quartile</td>
<td>1.33%</td>
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<tr>
<td>Cases with Profit Increase</td>
<td>4168</td>
<td>3476</td>
</tr>
<tr>
<td>Cases with Profit Decrease</td>
<td>782</td>
<td>1474</td>
</tr>
</tbody>
</table>

Table 1  Summary statistics on percentage difference in equilibrium profits for H and L over 4,950 test cases when H or L commits to static pricing.

To better understand how relative product quality affects the value of price commitment, we first present, in Figure 4, the average percentage difference in equilibrium profits when H (left) or L (right) commits to static pricing across all $\gamma$ values tested. When H commits, the left graph in Figure 4 shows that the average percentage difference tends to be small but positive for H, but is large and negative for L, except when $\beta$ is large. The right graph in Figure 4 shows that when L commits, the average percentage difference is positive for L for all $\beta$ values and positive for H as well, except for a very small $\beta$.

We also demonstrate the percentage difference in equilibrium profits for fixed values of $\gamma$ when H (left) or L (right) commits to static pricing. For example, Figure 5 shows the case of $\gamma = 0.2$. Note
Figure 5  Percentage difference in equilibrium profits when H (left) or L (right) commits to static pricing for \( \gamma = 0.2 \).

that for the fixed \( \gamma \) value, the percentage difference is increasing in \( \beta \) when H commits; see the left figure. When L commits, however, the percentage difference is decreasing for L while increasing for H. This suggests that the lower the quality of product L, the higher the profit loss for L when H commits, but the higher the profit gains when L commits.

Our main observation in this section is that the price commitments of H and L have very different effects. While the price commitment of L in general benefits both firms, the price commitment of H can benefit itself but hurts L. When a firm commits to static pricing, it will only incur sales in the first period since customers discount future utilities, and it is never beneficial for them to delay purchases from the firm that commits to static pricing. This implies that the other firm that dynamically changes prices is relieved from price competition against its competitor in the future. On the other hand, the price-commitment firm is able to extract more profits from high-valuation customers who are discouraged from waiting when facing static prices, thus leaving a market of low-value customers to its competitor. Therefore, whether the firm benefits from the price commitment of its competitor depends on which effect dominates. When the high-quality firm commits, the benefit from reducing future price competition among firms cannot compensate for the loss from sales shrinkage of high-valuation customers, leading to profit loss for the low-quality firm. On the contrary, when the low-quality firm commits, the high-quality firm is still able to secure a portion of high-valuation customers due to its quality advantage, and at the same time benefits from less pricing competition in future periods. Therefore, pricing competition between the two firms and strategic customer behavior interact in a subtle way in the case of unilateral price commitment, and quality plays a key role here.
<table>
<thead>
<tr>
<th>Firm</th>
<th>H</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>7.63%</td>
<td>7.10%</td>
</tr>
<tr>
<td>Minimum</td>
<td>-29.04%</td>
<td>-74.76%</td>
</tr>
<tr>
<td>Average</td>
<td>-3.92%</td>
<td>-22.09%</td>
</tr>
<tr>
<td>Third Quartile</td>
<td>-6.96%</td>
<td>-36.08%</td>
</tr>
<tr>
<td>Median</td>
<td>-2.47%</td>
<td>-17.42%</td>
</tr>
<tr>
<td>First Quartile</td>
<td>-0.09%</td>
<td>-5.44%</td>
</tr>
<tr>
<td>Cases with Profit Increase</td>
<td>1185</td>
<td>239</td>
</tr>
<tr>
<td>Cases with Profit Decrease</td>
<td>3864</td>
<td>4810</td>
</tr>
</tbody>
</table>

Table 2: Summary statistics on percentage difference in profits for H and L over 5,049 test cases when H and L simultaneously commit to static pricing.

5.3 Simultaneous Commitment to Static Pricing

What if both firms are able to commit to static pricing? In this case, the problem reduces to the last period one analyzed in section 3.1. We compute the percentage difference in equilibrium profits, \( \frac{r_{i}^{\text{H}}(\beta, \gamma) - r_{i}^{\text{L}}(\beta, \gamma)}{r_{i}^{\text{L}}(\beta, \gamma)} \times 100 \), for different combinations of \( \beta \) and \( \gamma \) over 5,049 cases. While there are cases where the gain from dynamic pricing is positive when \( \gamma \) is relatively small, dynamic pricing results in profit loss in the majority of cases for both H and L. Moreover, we observe that the percentage difference decreases as \( \gamma \) increases; that is, when customers are more strategic, dynamic pricing is less preferred.

Table 2 reports some summary statistics for the 5,049 cases we consider. The percentage profit loss for H is between 7.63% and -29.04%, with a mean of -3.92% and a median of -2.47%. Out of the 5,049 cases, the gain from dynamic pricing is positive for 1,185 cases. Therefore, even though dynamic pricing tends to decrease the profit for H, the average magnitude of the loss is relatively small. The statistics for L is in stark contrast to that for H. The percentage profit loss for L is between 7.10% and -74.76%, with a mean of -22.09% and a median of -17.42%. Out of 5,049 cases, only 239 cases have positive gains from dynamic pricing, and those cases with positive gains all have very small \( \gamma \) (i.e., customers are almost myopic). It is fair to say that dynamic pricing is, in general, worse than static pricing for L. Again, we see that the impact of strategic customer behavior is asymmetric for H and L, with the detrimental effects often orders of magnitude larger for L.

Figure 6 shows the average percentage difference in equilibrium profits across \( \gamma \) values for each \( \beta \) between dynamic pricing and static pricing for H and L. One salient observation is that H suffers less, on average, than L from dynamic pricing. Figure 7 shows the percentage difference in equilibrium profits between dynamic pricing and static pricing for H and L when \( \gamma = 0.2 \). Note that the relative loss (gain when \( \beta \) is small) is rather small for H, while the profit loss experienced by L is quite substantial, especially when two products are highly differentiated.
Our results suggest that in the presence of strategic customers, firms can gain from commitment to static pricing. We note, however, that a pre-announced single price policy is not subgame perfect. To credibly commit to static pricing, firms would need to use certain commitment devices, such as advertising or contracting; see, for example, Su and Zhang (2008).

5.4 The Impact of Pricing Opportunities

When all customers are myopic, Proposition 7 shows that firms always prefer a longer selling horizon with more pricing opportunities. Does it hold when customers are strategic? How do the equilibrium profits with strategic customers change with the number of pricing opportunities? Note that in our model, the number of pricing opportunities is the same as the number of periods $T$.  

**Figure 6** Average percentage difference in equilibrium profits between dynamic pricing and static pricing for H and L.

**Figure 7** Percentage difference in equilibrium profits between dynamic pricing and static pricing for H and L when $\gamma = 0.2$. 
<table>
<thead>
<tr>
<th>Firm</th>
<th>H</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>0.57%</td>
<td>0.47%</td>
</tr>
<tr>
<td>Minimum</td>
<td>-5.49%</td>
<td>-20.77%</td>
</tr>
<tr>
<td>Average</td>
<td>-0.53%</td>
<td>-2.27%</td>
</tr>
<tr>
<td>Third Quartile</td>
<td>-0.73%</td>
<td>-2.98%</td>
</tr>
<tr>
<td>Median</td>
<td>-0.16%</td>
<td>-0.60%</td>
</tr>
<tr>
<td>First Quartile</td>
<td>-0.01%</td>
<td>-0.05%</td>
</tr>
<tr>
<td>Cases with Profit Increase</td>
<td>588</td>
<td>239</td>
</tr>
<tr>
<td>Cases with Profit Decrease</td>
<td>4461</td>
<td>4810</td>
</tr>
</tbody>
</table>

Table 3  Summary statistics on percentage difference in equilibrium profits between the limiting equilibrium and the case of $T = 2$ for H and L over 5,049 test cases.

We calculate the percentage difference in equilibrium profits, \( \frac{r^*_i(\beta, \gamma) - r^2_i(\beta, \gamma)}{r^*_i(\beta, \gamma)} \times 100, \) for both H and L over 5,049 cases. The percentage difference in profits is between 0.57% and -5.49%, with a mean of -0.53% and a median of 0.16% for H, while it is between 0.47% and -20.77%, with a mean of -2.27% and a median of -0.60% for L. Therefore, the magnitude of difference is rather small overall for both H and L. Out of 5,049 cases, there are 588 and 239 cases where the difference is positive for H and L, respectively. We further observe that $\gamma$ is, in general, very small in those cases with positive differences. Hence, lengthening time horizon $T$ tends to decrease profits for both H and L in the presence of strategic customers.

Figure 8 shows the average percentage difference in equilibrium profits between the limiting equilibrium and the case of $T = 2$ for H and L. Note that the average percentage difference is negative for L for all $\beta$ values. The average percentage difference is negative for H, except when $\beta$ is very small. Figure 9 shows the the percentage difference in equilibrium profits between the limiting equilibrium and the case of $T = 2$ for H and L when $\gamma = 0.2$. Note that the difference in absolute value decreases in $\beta$ for both H and L, with the magnitude for H much smaller than that for L.

Our results are somewhat negative from the firms’ perspective. We have shown that dynamic pricing tends to reduce a firm’s profit when customers are strategic. The results in this section further indicate that the detrimental effects of dynamic pricing are almost fully realized, even when each firm changes price only once.

### 5.5 The Cost of Ignoring Strategic Customer Behavior

What if one firm incorrectly assumes that customers are myopic? A firm who assumes customers are myopic would take $\gamma = 0$, different from its true value, and price accordingly. We assume the other firm knows the correct value of $\gamma$. Both firms use the equilibrium prices, with the only difference
Figure 8  Average percentage difference in profits between the limiting equilibrium and the case of $T = 2$ for H and L.

Figure 9  Percentage difference in profits between the limiting equilibrium and the case of $T = 2$ for H and L when $\gamma = 0.2$.

being the \( \gamma \) values they use. Table 4 reports summary statistics over the 5,049 cases when either H or L assumes \( \gamma = 0 \). When H assumes \( \gamma = 0 \), the profit loss of H is between 0.00% and -5.36%, with an average of -0.33%, which is rather small. On the other hand, the profit gain of L is between 0.00% and 53.25%, with an average of 7.38%, which is much larger than the absolute loss of H.

When L assumes \( \gamma = 0 \), the profit gain for H is between 0.00% to 33.91%, with an average of 9.93%, which is rather significant. The profit loss for L is between 0.00% and -100.00% with an average of -22.53%, which is also very significant.

These results demonstrate that when a firm wrongly assumes myopic customers (\( \gamma = 0 \)), and its competitor uses the correct \( \gamma \) value, it incurs profit loss, while the competitor enjoys profit lift. As for the magnitude of profit loss (lift), it is closely related to relative quality level. See Figure 10
Table 4  Summary statistics on percentage loss in equilibrium profits when H or L assumes $\gamma = 0$.

<table>
<thead>
<tr>
<th>Firm</th>
<th>H assumes $\gamma = 0$</th>
<th>L assumes $\gamma = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
<td>L</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.00%</td>
<td>53.25%</td>
</tr>
<tr>
<td>Minimum</td>
<td>-5.36%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Average</td>
<td>-0.33%</td>
<td>7.98%</td>
</tr>
<tr>
<td>Third Quartile</td>
<td>-0.37%</td>
<td>1.84%</td>
</tr>
<tr>
<td>Median</td>
<td>-0.08%</td>
<td>5.17%</td>
</tr>
<tr>
<td>First Quartile</td>
<td>-0.01%</td>
<td>11.38%</td>
</tr>
</tbody>
</table>

Figure 10  Average percentage difference in equilibrium profits when H (left) or L (right) assumes $\gamma = 0$.

and Figure 11 for illustration.

Figure 10 shows the average percentage loss in equilibrium profits when H (left) or L (right) incorrectly assumes $\gamma = 0$. When firm H ignores the strategic purchase behavior of customers, its profit is reduced, but to a moderate degree. However, its competitor, firm L, enjoys a large magnitude of profit lift in general. When L wrongly assumes that $\gamma = 0$, it suffers substantially in general, but its competitor gains significantly. Figure 11 plots the percentage profit loss for a fixed value $\gamma$. We notice that both loss and gain in absolute value are decreasing in $\beta$, implying that the effect is more dramatic when two firms differ largely in quality level.

Taken together, the results suggest that H tends to suffer much less on average than L (-0.33% vs. -22.53%) from using an incorrect $\gamma$ value. A similar observation has also been made by Levin et al. (2009). One implication is therefore that it is much more important for the low-quality firm to manage strategic customer behavior. It is also interesting to note that when one firm assumes an incorrect value for $\gamma$, it suffers but its competitor gains. This, in a way, suggests that firms would be unwilling to share information on strategic customer behavior.
6. Summary

We consider the dynamic pricing competition between two vertically differentiated firms when customers are strategic. We show that there exists an MPE in mixed strategy in the game. Furthermore, we give a simple condition for the existence of a unique pure-strategy MPE, which admits explicit recursive expressions. In particular, this result immediately implies the existence of a pure-strategy MPE when customers are myopic. Our results yield the following insights. We show that the presence of strategic customer behavior leads to qualitatively different results regarding the value of dynamic pricing. While dynamic pricing always increases a firm’s profit when customers are myopic, it is often undesirable when customers are strategic. This result can be viewed as an extension from a monopoly market in Besanko and Winston (1990) to a competitive setting. More importantly, the impact of strategic customer behavior is asymmetric for quality-differentiated firms, with the low-quality firm suffering much more than the high-quality firm; in this sense, the disadvantage of offering inferior products is exacerbated by strategic customer behavior. Furthermore, the detrimental effects of dynamic pricing are almost fully realized when each firm only reduces price once. These results are somewhat negative from a practical standpoint, implying that firms should be careful when adopting dynamic pricing strategy, even when prices are changed infrequently.

Motivated by the results regarding dynamic pricing competition, we study versions of the game where one firm commits to static pricing (charging a single price over the entire selling horizon), while the other firm dynamically changes prices over time. We show that a unilateral commitment to static pricing usually benefits the committing firm. Moreover, it can also improve the profits of both firms when the low-quality firm makes such a commitment. Therefore, static pricing strategies can often be justified when customers behave strategically.
The stylized model we consider follows closely the modeling framework of Besanko and Winston (1990) and extends their work to competitive settings. In addition to reinforcing some of their analytical results and insights, our model also generates novel insights on the effect of quality differentiation and price commitment. While we do believe that relaxing some of our assumptions is of general interest and can sharpen our insights, our initial attempt suggests that it will often lead to significant analytical challenges, and therefore, constitutes interesting future research directions.

Acknowledgment

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References


Appendix

Proof of Proposition 1.

When \( p_{T,H} \geq p_{T,L} + (1 - \beta)\theta_T \) or \( p_{T,H} < \frac{p_{T,L}}{\beta} \), one of the firms receives 0 profit; therefore, a Nash equilibrium cannot be sustained. This means that a Nash equilibrium price pair must satisfy \( \frac{p_{T,L}}{\beta} \leq p_{T,H} < p_{T,L} + (1 - \beta)\theta_T \), in which case the first-order conditions lead to the price pair given in (2). The corresponding profits are given by (3).

To show that the price pair \( (p_{T,H}^*(\theta_T), p_{T,L}^*(\theta_T)) \) is a Nash equilibrium, we need to show that both firms do not have incentive to deviate from this solution. First observe that \( p_{T,H}^*(\theta_T) < (1 - \beta)\theta_T + \beta c \); hence, firm L cannot deviate to improve his profit. For firm H, it can be shown that the optimal price in the region \( p_{T,H} \leq \frac{p_{T,L}(\theta_T)}{\beta} \) is given by \( \frac{p_{T,L}(\theta_T)}{\beta} = \frac{(1 - \beta)(\theta_T - c)}{4 - \beta} + c \) with corresponding profit \( \frac{3(1 - \beta)(\theta_T - c)^2}{(4 - \beta)^2} < r_{1,H}^*(\theta_T) \). Thus, firm H also does not have an incentive to deviate. This establishes the price pair \( (p_{T,H}^*(\theta_T), p_{T,L}^*(\theta_T)) \) as the unique Nash equilibrium in the last period. This completes the proof.

Proof of Proposition 2

Proposition 2 is a special case of Proposition 3.
Proof of Theorem 1

The proof is by backward induction. The result holds for the last period from Proposition 1. Fix $t < T$ and suppose the remaining customers have valuations in the range $[0, \theta_t]$. Suppose there exists an MPE for the game starting at time $t + 1$ with customer valuations in the range $[0, \theta_{t+1}]$. Assume that a mixed-strategy equilibrium profile in period $t + 1$ is given by $(\delta_{t+1,H}(\theta_{t+1}, p_{t,H}), \delta_{t+1,L}(\theta_{t+1}, p_{t,L}))$, where $\delta_{t+1,H}(\theta_{t+1}, \cdot)$ is a probability density function on $[c, \theta_{t+1}]$ and $\delta_{t+1,L}(\theta_{t+1}, \cdot)$ is a probability density on $[\beta c, \theta_{t+1}]$. Given price pair $p_t = (p_{t,H}, p_{t,L})$, let $\hat{\theta}$ be the cutoff value below which customers do not purchase in period $t$. If $L$ has positive demand in period $t$, then $\hat{\theta}$ is a function of $p_{t,L}$, $\delta_{t+1,H}$, and $\delta_{t+1,L}$; if $L$ has no demand in period $t$, then $\hat{\theta}$ is a function of $p_{t,H}$, $\delta_{t+1,H}$, and $\delta_{t+1,L}$. To accommodate both of these cases, we adopt the general notation $\hat{\theta}(p_{t,H}, \delta_{t+1,H}, \delta_{t+1,L})$. Then the payoff functions of the two players can be written as

$$
egin{align*}
  r_{t,H}(\theta_t, p_t) &= \begin{cases} 
  (p_{t,H} - c) \left[ \theta_t - \frac{p_{t,H} - p_{t,L}}{\frac{p_{t,H} - p_{t,L}}{1 - \gamma}} \right]^+ + \alpha r_{t+1,H}^+ \left( \hat{\theta}(p_{t,H}, \delta_{t+1,H}, \delta_{t+1,L}) \right), & \text{if } L \text{ has positive demand in period } t, \\
  (p_{t,H} - c) \left[ \theta_t - \hat{\theta}(p_{t,H}, \delta_{t+1,H}, \delta_{t+1,L}) \right]^+ + \alpha r_{t+1,H}^+ \left( \hat{\theta}(p_{t,H}, \delta_{t+1,H}, \delta_{t+1,L}) \right), & \text{if } L \text{ has no demand in period } t, 
  \end{cases}
\end{align*}
$$

$$
egin{align*}
  r_{t,L}(\theta_t, p_t) &= \begin{cases} 
  (p_{t,L} - \beta c) \left[ \frac{p_{t,H} - p_{t,L}}{1 - \gamma} - \hat{\theta}(p_{t,L}, \delta_{t+1,H}, \delta_{t+1,L}) \right]^+ + \alpha r_{t+1,L}^+ \left( \hat{\theta}(p_{t,L}, \delta_{t+1,H}, \delta_{t+1,L}) \right), & \text{if } L \text{ has positive demand in period } t, \\
  \alpha r_{t+1,L}^+ \left( \hat{\theta}(p_{t,L}, \delta_{t+1,H}, \delta_{t+1,L}) \right), & \text{if } L \text{ has no demand in period } t. 
  \end{cases}
\end{align*}
$$

First note that the equilibrium payoff functions $r_{t+1,H}^+ (\cdot)$ and $r_{t+1,L}^+ (\cdot)$ are continuous since the argument is a scale parameter. Furthermore, it can be shown that the cutoff value $\hat{\theta}(p_{t,H}, \delta_{t+1,H}, \delta_{t+1,L})$ is continuous in $p_{t,H}$. Therefore, the functions $r_{t,H}(\theta_t, p_t)$ and $r_{t,L}(\theta_t, p_t)$ are continuous. The strategy space $[c, \theta_t] \times [\beta c, \beta \theta_t]$ is nonempty and compact. The result follows by an application of Theorem 3 in Dasgupta and Maskin (1986); see also Glicksberg (1952).

Proof of Proposition 3

(i) Note that

$$
\beta - \gamma(1 - A_{t+1,H}) - 2\alpha B_{t+1,L} > 0,
$$

implies that

$$
\beta - \gamma(1 - A_{t+1,H}) > 0
$$

since $B_{t+1,L} > 0$. The strategy space $[c, \theta_t] \times [\beta c, \beta \theta_t]$ can be divided into several regions where different product combinations are offered. Let $\hat{\theta}$ be the valuation of the marginal customer who is
indifferent between purchasing in period $t$ and period $t+1$. Then the strategy space can be divided into four regions (see Figure 12).

Region I: L only. If $\theta - p_{t,H} \leq \beta \theta - p_{t,L}$ for all $\theta \in [\hat{\theta}, \theta_t]$, i.e., $p_{t,H} \geq p_{t,L} + (1 - \beta) \theta_t$, then no customer will purchase product $H$ in period $t$. The marginal valuation $\hat{\theta}$ satisfies $\beta \hat{\theta} - p_{t,L} = \gamma (\hat{\theta} - p_{t+1,H}(\hat{\theta}))$. Here we assume product $H$ is offered in period $t$, which is true by inductive assumption. It follows that $\hat{\theta} \leq \theta_t$, which implies this case is possible only when $p_{t,L} \leq \beta \theta_t - \gamma (1 - A_{t+1,H})(\theta_t - c)$.

Region II: H only. If $\theta - p_{t,H} \geq \beta \theta - p_{t,L}$ for all $\theta$ in $[\hat{\theta}, \theta_t]$, then no customer will purchase product $L$ in period $t$. Note that the marginal valuation $\hat{\theta}$ satisfies $\hat{\theta} - p_{t,L} = \gamma (\hat{\theta} - p_{t+1,H}(\hat{\theta}))$. Here again, we assume product $H$ is offered in period $t$, which is true by inductive assumption. It follows that $\hat{\theta} = \frac{p_{t,L} - \gamma (1 - A_{t+1,H})c}{\beta - \gamma (1 - A_{t+1,H})}$. Furthermore, since customers in period $t$ only purchase product $H$, we must have $\frac{p_{t,H} - p_{t,L}}{1 - \beta} \leq \hat{\theta}$, which can be reduced to $p_{t,H} \leq \frac{[1 - \gamma (1 - A_{t+1,H})](p_{t,L} - \beta c)}{\beta - \gamma (1 - A_{t+1,H})} + c$. Note that we need to have $\hat{\theta} \leq \theta_t$, which implies this case is possible only when $p_{t,H} \leq \beta \theta_t - \gamma (1 - A_{t+1,H})(\theta_t - c)$.

Region III: Both H and L. When $\frac{[1 - \gamma (1 - A_{t+1,H})](p_{t,L} - \beta c)}{\beta - \gamma (1 - A_{t+1,H})} + c < p_{t,H} < p_{t,L} + (1 - \beta) \theta_t$, then $p_{t,L} < \beta \theta_t - \gamma (1 - A_{t+1,H})(\theta_t - c)$ and $p_{t,H} < \theta_t - \gamma (1 - A_{t+1,H})(\theta_t - c)$, both firms face positive demand. The marginal valuation is given by $\hat{\theta} = \frac{p_{t,L} - \gamma (1 - A_{t+1,H})c}{1 - \gamma (1 - A_{t+1,H})}$.

Figure 12  Regions of strategy space in period $t$. 
Region IV: Zero demand for both products. When $p_{t,L} > \beta \hat{\theta}_t - \gamma(1 - A_{t+1,H})(\theta_t - c)$ and $p_{t,H} > \theta_t - \gamma(1 - A_{t+1,H})(\theta_t - c)$, the demand for each product is 0.

The payoff function of firm $H$ is

$$r_{t,H}(\theta_t, p_t) = \begin{cases} 
\alpha r_{t+1,H}^* \left( \frac{p_{t,L} - \gamma(1 - A_{t+1,H})c}{\beta - \gamma(1 - A_{t+1,H})} \right), & \text{if } p_t \text{ in region I,} \\
(p_{t,H} - c) \left( \theta_t - \frac{p_{t,H} - \gamma(1 - A_{t+1,H})c}{1 - \gamma(1 - A_{t+1,H})} \right) + \alpha r_{t+1,H}^* \left( \frac{p_{t,L} - \gamma(1 - A_{t+1,H})c}{\beta - \gamma(1 - A_{t+1,H})} \right), & \text{if } p_t \text{ in region II,} \\
(p_{t,H} - c) \left( \theta_t - \frac{p_{t,H} - p_t L}{1 - \beta} \right) + \alpha r_{t+1,H}^* \left( \frac{p_{t,L} - \gamma(1 - A_{t+1,H})c}{\beta - \gamma(1 - A_{t+1,H})} \right), & \text{if } p_t \text{ in region III,} \\
\alpha r_{t+1,H}^* (\hat{\theta}_t), & \text{if } p_t \text{ in region IV.} 
\end{cases}$$

Similarly, the payoff function of firm $L$ is

$$r_{t,L}(\theta_t, p_t) = \begin{cases} 
(p_{t,L} - \beta c) \left( \theta_t - \frac{p_{t,L} - \gamma(1 - A_{t+1,H})c}{\beta - \gamma(1 - A_{t+1,H})} \right) + \alpha r_{t+1,L}^* \left( \frac{p_{t,L} - \gamma(1 - A_{t+1,H})c}{\beta - \gamma(1 - A_{t+1,H})} \right), & \text{if } p_t \text{ in region I,} \\
(p_{t,L} - \beta c) \left( \frac{p_{t,H} - p_t L}{1 - \beta} \right) - \frac{p_{t,L} - \gamma(1 - A_{t+1,H})c}{\beta - \gamma(1 - A_{t+1,H})} + \alpha r_{t+1,L}^* \left( \frac{p_{t,L} - \gamma(1 - A_{t+1,H})c}{\beta - \gamma(1 - A_{t+1,H})} \right), & \text{if } p_t \text{ in region II,} \\
(p_{t,L} - \beta c) \left( \frac{p_{t,L} - p_t L}{1 - \beta} \right) - \frac{p_{t,L} - \gamma(1 - A_{t+1,H})c}{\beta - \gamma(1 - A_{t+1,H})} + \alpha r_{t+1,L}^* \left( \frac{p_{t,L} - \gamma(1 - A_{t+1,H})c}{\beta - \gamma(1 - A_{t+1,H})} \right), & \text{if } p_t \text{ in region III,} \\
\alpha r_{t+1,L}^* (\hat{\theta}_t), & \text{if } p_t \text{ in region IV.} 
\end{cases}$$

Now we show that there exists a Nash equilibrium in period $t$ in Region III. The proof proceeds in several steps. We first solve the first order conditions in each region and show that these solutions are valid. We then show that the solution in Region III is a Nash equilibrium by demonstrating that both firms have no incentive to deviate from this solution, satisfying the definition of Nash equilibrium. Finally, we show that the solutions in regions I, II, and IV do not satisfy Nash equilibrium conditions, establishing the uniqueness of the equilibrium.

Step 1: Solve the the first-order conditions in each region.

We first solve the first-order conditions in each region, ignoring the boundary conditions. We obtain the following solutions for $H$ and $L$, respectively:

$$\hat{p}_{t,H}(\theta_t) = \begin{cases} 
\text{any value in range,} & \text{if solution in region I,} \\
\frac{[\beta - \gamma(1 - A_{t+1,H})]2(\theta_t - c)}{2[\beta - \gamma(1 - A_{t+1,H}) - \alpha B_{t+1,H}]} + c, & \text{if solution in region II,} \\
\alpha r_{t+1,H}^* \left( \frac{p_{t,L} - \gamma(1 - A_{t+1,H})c}{\beta - \gamma(1 - A_{t+1,H})} \right), & \text{if solution in region III,} \\
\text{any value in range,} & \text{if solution in region IV,}
\end{cases}$$

$$\hat{p}_{t,L}(\theta_t) = \begin{cases} 
\text{any value in range,} & \text{if solution in region I,} \\
\frac{[\beta - \gamma(1 - A_{t+1,H})]2(\theta_t - c)}{2[\beta - \gamma(1 - A_{t+1,L}) - \alpha B_{t+1,L}]} + \beta c, & \text{if solution in region II,} \\
\frac{[\beta - \gamma(1 - A_{t+1,L})]2(\theta_t - c)}{3[\beta - \gamma(1 - A_{t+1,L})]2 + 4(1 - \beta)[\beta - \gamma(1 - A_{t+1,L})] - 4\alpha(1 - \beta)B_{t+1,L}} + \beta c, & \text{if solution in region III,} \\
\text{any value in range,} & \text{if solution in region IV,}
\end{cases}$$
Next, we show these solutions from first-order conditions are in their respective regions, and are therefore valid. Note that we only need to check for regions I, II, and III.

To check whether the solution is valid in region I, we need to show

\[
\hat{p}_{t,L}(\theta_t) = \frac{[\beta - \gamma(1 - A_{t+1,H})]^2(\theta_t - c)}{2[\beta - \gamma(1 - A_{t+1,H}) - \alpha B_{t+1,L}]} + \beta c \leq [\beta - \gamma(1 - A_{t+1,H})](\theta_t - c) + \beta c.
\]

In the above, the inequality is equivalent to (35).

To check whether the solution is valid in region II, we need to show

\[
\hat{p}_{t,H}(\theta_t) = \frac{[1 - \gamma(1 - A_{t+1,H})]^2(\theta_t - c)}{2[1 - \gamma(1 - A_{t+1,H}) - \alpha B_{t+1,H}]} + c \leq [1 - \gamma(1 - A_{t+1,H})](\theta_t - c) + c,
\]

which requires

\[
1 - \gamma(1 - A_{t+1,H}) - 2\alpha B_{t+1,H} \geq 0.
\]

Note that

\[
1 - \gamma(1 - A_{t+1,H}) - 2\alpha B_{t+1,H} = 1 - \beta + \beta - \gamma(1 - A_{t+1,H}) - 2\alpha B_{t+1,H} \geq 1 - \beta + 2\alpha B_{t+1,L} - 2\alpha B_{t+1,H} \geq 0,
\]

where the first inequality follows from (35), and the second inequality follows from $B_{t+1,H} - B_{t+1,L} \leq \frac{1-\beta}{2}$.

To check whether the solution is valid in region III, we first note that region III is defined by the following inequalities:

\[
\begin{align*}
\hat{p}_{t,L}(\theta_t) & \leq [\beta - \gamma(1 - A_{t+1,H})](\theta_t - c) + \beta c, & \text{(38)} \\
\hat{p}_{t,H}(\theta_t) & \leq [1 - \gamma(1 - A_{t+1,H})](\theta_t - c) + c, & \text{(39)} \\
p_{t,H} & \leq (1 - \beta)\theta_t + p_{t,L}, & \text{(40)} \\
p_{t,H} & \geq \frac{[1 - \gamma(1 - A_{t+1,H})](p_{t,L} - \beta c)}{\beta - \gamma(1 - A_{t+1,H})} + c. & \text{(41)}
\end{align*}
\]

We will check to see that inequalities (38)–(41) are satisfied by the solution $(\hat{p}_{t,H}(\theta_t), \hat{p}_{t,L}(\theta_t))$.

First, using (35) in the denominator, we have

\[
\hat{p}_{t,L}(\theta_t) \leq \frac{(1 - \beta)[\beta - \gamma(1 - A_{t+1,H})]^2(\theta_t - c)}{[\beta - \gamma(1 - A_{t+1,H})]^2 + 2(1 - \beta)[\beta - \gamma(1 - A_{t+1,H})]} + \beta c \\
\leq \frac{1}{2}[\beta - \gamma(1 - A_{t+1,H})](\theta_t - c) + \beta c.
\]

Therefore, (38) is satisfied. Using (38) and the expression for $\hat{p}_{t,H}(\theta_t)$, we can show that (39) and (40) are satisfied. To check whether (41) holds, note that

\[
\hat{p}_{t,H}(\theta_t) - \frac{[1 - \gamma(1 - A_{t+1,H})](p_{t,L} - \beta c)}{\beta - \gamma(1 - A_{t+1,H})} - c
\]
\[
2(1-\beta)\left[\beta - \gamma (1-A_{t+1,H})\right]^2(\theta_t-c)\left[\beta - \gamma (1-A_{t+1,H})\right]^2 + (1-\beta)\left[\beta - \gamma (1-A_{t+1,H})\right] - 2\alpha(1-\beta)B_{t+1,L} \geq 0.
\]

In the above, the inequality follows by applying (35).

**Step 2: show that the solution in region III is a Nash equilibrium.**

To check whether the solution in range III is an NE, we need to show that neither firm has an incentive to deviate. We first show that firm L has no incentive to deviate. Note that firm L cannot increase profit by deviating to region II where only demand for H is positive in period \(t\). This follows immediately from the fact that \(\hat{p}_{t,L}(\theta_t)\) is the optimal solution for L in range III, which is necessarily better than the solution on the boundary of range III and range II when \(\hat{p}_{t,H}(\theta_t)\) stays constant. Now, we show that L cannot deviate to range I since \(\hat{p}_{t,H}(\theta_t)\leq (1-\beta)\theta_t + \beta c\). To show this, note that by inequality (43), we have

\[
\hat{p}_{t,L}(\theta_t) \leq \frac{(1-\beta)\left[\beta - \gamma (1-A_{t+1,H})\right]^2(\theta_t-c) + \beta c \leq (1-\beta)(\theta_t-c) + \beta c.}
\]

It follows that

\[
\hat{p}_{t,L}(\theta_t) = \frac{\hat{p}_{t,L}(\theta_t) + (1-\beta)\theta_t + c}{2} \leq (1-\beta)\theta_t + \beta c.
\]

We next show that firm H has no incentive to deviate. First, it is immediate that firm H will not deviate to range I since the marginal valuation \(\hat{\theta}\) is determined by the price of L in both range I and range III. By deviating from range III to range I, firm H earns no profit in period \(t\) and the same future profit. Next we show that firm H will not deviate from range III to range II. By the optimality of \(\hat{p}_{t,H}(\theta_t)\) in range III, firm H has no incentive to deviate from \(\hat{p}_{t,H}(\theta_t)\) to the boundary of range II and range III when the price of L stays constant. Now in region II, the price of H determined by a first-order condition is given by

\[
\hat{p}_{t,H}(\theta_t) = \frac{[1-\gamma(1-A_{t+1,H})]^{2(\theta_t-c)}}{2(1-\gamma(1-A_{t+1,H})-\alpha\hat{B}_{t+1,H})} + c \geq \frac{[1-\gamma(1-A_{t+1,H})]^{(\theta_t-c)}}{2} + c.\]

On the other hand,

\[
\hat{p}_{t,L}(\theta_t) \leq \frac{(1-\beta)\left[\beta - \gamma (1-A_{t+1,H})\right]^2(\theta_t-c) + \beta c \leq \frac{[\beta - \gamma (1-A_{t+1,H})](\theta_t-c)}{2} + \beta c.}
\]

It follows that

\[
\hat{p}_{t,H}(\theta_t) \geq \frac{[1-\gamma(1-A_{t+1,H})](\hat{p}_{t,L}(\theta_t)-\beta c)}{\beta - \gamma (1-A_{t+1,H})} + c.
\]

Therefore, the point \((\hat{p}_{t,H}(\theta_t),\hat{p}_{t,L}(\theta_t))\) lies above range II, implying that the profit of firm H is optimized at the boundary of range II and range III within range II. Combining the arguments above shows that firm H has no incentive to deviate from range III to range II.

**Step 3: show that solutions in regions I, II, and IV do not satisfy Nash equilibrium conditions.**
To show that the solution in region I cannot be a Nash equilibrium in period \( t \), we note that \( H \) does not have demand in region I. Therefore, \( H \) can always deviate to region III by lowering its price. Since the cutoff value for \( \theta \) (i.e., the number of customers remaining in the system) is determined by the price of \( L \), deviating to region III leads to higher profit in period \( t \) and the same future profit, and is therefore a profitable deviation for \( H \).

To show that the solution in region II cannot be a Nash equilibrium, we show that \( L \) can deviate profitably. First observe that \( L \) obtains no profit in region II where we must have \( \hat{p}_{t,L}(\theta_t) \geq \frac{[\beta - \gamma(1 - A_{t+1,H})]p_{t,H}(\theta_t) - c}{1 - \gamma(1 - A_{t+1,H})} \). Suppose \( L \) deviates while the price of \( H \) is held at \( \hat{p}_{t,H}(\theta_t) \). The optimal response for \( L \) is given by

\[
\tilde{p}_{t,L}(\theta_t) = \frac{[\beta - \gamma(1 - A_{t+1,H})]^2(\hat{p}_{t,H}(\theta_t) - c) - 2(\beta - \gamma(1 - A_{t+1,H})) - 2\alpha(1 - \beta)B_{t+1,L} + \beta c}{2[1 - \gamma(1 - A_{t+1,H})][\beta - \gamma(1 - A_{t+1,H})]}.
\]

The optimal response intersects the boundary of region II and region III at the point \( \left( \hat{p}_{t,H}(\theta_t), \frac{[\beta - \gamma(1 - A_{t+1,H})](\hat{p}_{t,H}(\theta_t) - c)}{1 - \gamma(1 - A_{t+1,H})} + \beta c \right) \). Applying (43), it can be shown that \( \tilde{p}_{t,L}(\theta_t) < \frac{[\beta - \gamma(1 - A_{t+1,H})](\hat{p}_{t,H}(\theta_t) - c)}{1 - \gamma(1 - A_{t+1,H})} + \beta c - \epsilon \) for a small positive constant \( \epsilon \).

To show that the solution in region IV cannot be a Nash equilibrium, we note the best response of \( L \) in region IV is given by the solution in region I, implying that \( L \) can always profitably deviate from region IV to region I.

Summarizing the arguments above shows part (i).

(ii) Since \( \beta - \gamma(1 - A_{t+1,H}) \leq 0 \), \( L \) has no demand in period \( t \) without regard to its price. The payoff function for both players take the form in region II for both players as in the proof of part (i). The Nash equilibrium follows immediately.

(iii) The result follows directly from Theorem 1.

Proof of Theorem 2

We use backward induction to establish a unique pure-strategy MPE in periods 1 to \( T - 1 \). The expressions for the last period follows from Proposition 1. Suppose the equilibrium prices and profits in period \( t + 1 \) when the remaining customers have valuations in the range \([0, \theta_{t+1}]\) are given by

\[
p^*_{t+1,H}(\theta_{t+1}) = A_{t+1,H}(\theta_{t+1} - c) + c, p^*_{t+1,L}(\theta_{t+1}) = A_{t+1,L}(\theta_{t+1} - c) + \beta c, r^*_{t+1,H}(\theta_{t+1}) = B_{t+1,H}(\theta_{t+1} - c)^2, r^*_{t+1,L}(\theta_{t+1}) = B_{t+1,L}(\theta_{t+1} - c)^2.
\]
Now we consider the game in period \( t \). Suppose the remaining customers in period \( t \) have valuations in the range \([0, \theta_t]\). From Lemma 2, (35) holds when \( \gamma \leq \beta \). Also the condition \( B_{t+1,H} - B_{t+1,L} \leq \frac{1-\beta}{2} \) holds when \( \gamma \leq \beta \) from Lemma 3. The result follows from part (i) of Proposition 3.

**Lemma 2.** Suppose \( \gamma \leq \beta \), the inequality

\[
\beta A_{t,H} - 2\alpha B_{t,L} \geq 0
\]

holds for all \( t \). In particular, for all \( t \), the following inequalities hold:

\[
\beta - \gamma(1 - A_{t,H}) - 2\alpha B_{t,L} \geq 0,
\]

\[
[\beta - \gamma(1 - A_{t,H})]^2 + (1 - \beta)[\beta - \gamma(1 - A_{t,H})] - 2\alpha(1 - \beta)B_{t,L} \geq 0.
\]

**Proof of Lemma 2.**

First note that when (42) holds, it is straightforward to show that (43) and (44) hold. We next show by induction that (42) holds. For \( t = T \), we have \( A_{T,H} = \frac{2(1-\beta)}{4-\beta} \) and \( B_{T,L} = \frac{\beta(1-\beta)}{(4-\beta)^2} \). Therefore the inequality holds for \( t = T \). Now suppose the inequality holds in period \( t + 1 \). We have

\[
\frac{A_{t,H}}{B_{t,L}} = \frac{2\left(3[\beta - \gamma(1 - A_{t+1,H})]^2 + 4(1 - \beta)[\beta - \gamma(1 - A_{t+1,H})] - 4\alpha(1 - \beta)B_{t+1,L}\right)}{[\beta - \gamma(1 - A_{t+1,H})]^2 + 2(1 - \beta)[\beta - \gamma(1 - A_{t+1,H})] - 2\alpha(1 - \beta)B_{t+1,L}}
\]

\[
\geq \frac{2\left(3 + 2(1 - \beta)(\beta - \gamma(1 - A_{t+1,H}))\right)}{\beta - \gamma(1 - A_{t+1,H})} - \frac{2\alpha(1 - \beta)B_{t+1,L}}{[\beta - \gamma(1 - A_{t+1,H})]^2}
\]

\[
= \frac{2(2 + \beta)}{\beta}
\]

This verifies that (42) holds in period \( t \). In the above, the first inequality follows by applying (44).

This completes the proof of the lemma.

**Lemma 3.** Suppose \( \gamma \leq \beta \), then \( B_{t,H} - B_{t,L} \leq \frac{1-\beta}{2} \) for all \( t \).

**Proof.** The proof is by induction. It can be checked that the inequality holds for \( T \). Suppose the inequality holds for period \( t + 1 \). We have

\[
B_{t,H} - B_{t,L} = \frac{(1 - \beta)}{4\Delta_{t+1}^2}\left(\Delta_{t+1} + (\beta - \gamma(1 - A_{t+1,H}))\right) + 4\alpha(1 - \beta)B_{t+1,H}(\beta - \gamma(1 - A_{t+1,H}))^2
\]

\[-2\Delta_{t+1}(\beta - \gamma(1 - A_{t+1,H}))^2 + 2(\beta - \gamma(1 - A_{t+1,H}))^4
\]
\[ + 4(1 - \beta)(\beta - \gamma(1 - A_{t+1,H}))^3 - 4\alpha(1 - \beta)B_{t+1,L}(\beta - \gamma(1 - A_{t+1,H}))^2 \]
\[ = \frac{(1 - \beta)}{4\Delta^2_{t+1}} \left( \Delta^2_{t+1} + 3(\beta - \gamma(1 - A_{t+1,H}))^4 + 4(1 - \beta)(\beta - \gamma(1 - A_{t+1,H}))^3 
+ 3(\beta - \gamma(1 - A_{t+1,H}))^4 + 4(1 - \beta)(\beta - \gamma(1 - A_{t+1,H}))^3 
+ 2(1 - \beta)^2(\beta - \gamma(1 - A_{t+1,H}))^2 \right). \]

In the above, the last inequality follows by the inductive assumption. To show that the inequality holds in period \( t \), it suffices to show that the second term above is less than \((1 - \beta)/4\), where it is equivalent to

\[ \left( \frac{\beta - \gamma(1 - A_{t+1,H})}{\Delta^2_{t+1}} \right)^2 \left( 3(\beta - \gamma(1 - A_{t+1,H}))^2 + 4(1 - \beta)(\beta - \gamma(1 - A_{t+1,H})) + 2\alpha(1 - \beta)^2 \right) \leq 1. \]

The inequality above is equivalent to

\[ 7(\beta - \gamma(1 - A_{t+1,H}))^4 + 8(1 - \beta)(\beta - \gamma(1 - A_{t+1,H}))^3 
+ 12(1 - \beta)(\beta - \gamma(1 - A_{t+1,H}))^2[\beta - \gamma(1 - A_{t+1,H}) - 2\alpha B_{t+1,L}] 
+ 16(1 - \beta)^2[\beta - \gamma(1 - A_{t+1,H}) - \alpha B_{t+1,L}]^2 \geq 0. \]

The inequality holds because the third term on the left-hand side is nonnegative by Lemma 2. This completes the proof of the lemma. \( \blacksquare \)

**Proof of Proposition 4**

By Lemma 4,

\[ \frac{A_{t,H}}{A_{t,L}} \geq \frac{A_{t+1,H}}{\beta A_{t+1,H}} \geq \frac{A_{t+1,H}}{\beta - \gamma(1 - A_{t+1,H})}. \]

Then

\[ A_{t,H}(\theta_{t+1} - c) \geq A_{t+1,H} \frac{A_{t,L}(\theta_{t+1} - c)}{\beta - \gamma(1 - A_{t+1,H})}. \]

Note that the period-\( t \) marginal customer, \( \theta_t^* \), is determined by:

\[ \theta_t^* = \frac{n_t^* - \gamma(1 - A_{t+1,H})c}{\beta - \gamma(1 - A_{t+1,H})} \]
Hence, \( A_{t,H}(\theta_{t+1} - c) \geq A_{t+1,H}(\theta_t^* (\theta_{t+1} - c)) \), which implies that \( p_{t,H}^* \geq p_{t+1,H}^* \). We next show that \( p_{t,L}^* \geq p_{t+1,L}^* \). Since \( \beta - \gamma(1 - A_{t+1,H}) \geq \beta A_{t+1,H} \geq A_{t+1,L} \), we have

\[
\frac{A_{t,L}(\theta_{t+1} - c)}{\beta - \gamma(1 - A_{t+1,H})} = A_{t+1,L}(\theta_t^* (\theta_{t+1} - c)).
\]

Therefore, \( p_{t,L}^* \geq p_{t+1,L}^* \).

\[\text{Lemma 4.} \quad \beta A_{t,H} \geq A_{t,L}, \quad \forall T \geq t \geq 1.\]

\text{Proof.} By Theorem 2, we have

\[
\frac{A_{t,H}}{A_{t,L}} = \frac{2[(\beta - \gamma(1 - A_{t+1,H})]^2 + (1 - \beta)[\beta - \gamma(1 - A_{t+1,H})] - \alpha(1 - \beta)B_{t+1,L}]}{[\beta - \gamma(1 - A_{t+1,H})]^2 + (1 - \beta)[\beta - \gamma(1 - A_{t+1,H})]}
\]

\[
\geq \frac{1 - \gamma(1 - A_{t+1,H})}{\beta - \gamma(1 - A_{t+1,H})} = \frac{1}{\beta}.
\]

The first inequality follows by applying (44).

\[\text{Proof of Proposition 5}\]

First note that by Theorem 2, \( A_{t,H} \) and \( A_{t,L} \) approaches 0 as \( \beta \) goes to 1. This implies that the total profit approaches 0 when \( \beta \) goes to 1, i.e., \( B_{t,H} \) and \( B_{t,L} \) go to 0. This completes the proof.

\[\text{Proof of Proposition 7}\]

We first show that \( \tilde{B}_{t+1,L} \geq \tilde{B}_{t+2,L} \), for \( T - 1 \geq t \geq 1 \). We prove the result by induction. Since \( \tilde{B}_{T,L} = \frac{\beta(1-\beta)}{(4-\beta)^2} \), we can check that

\[
\tilde{B}_{T-1,L} = \frac{\beta(1-\beta)(1 - \frac{\alpha(1-\beta)}{(4-\beta)^2})}{(4 - \beta - \frac{4\alpha(1-\beta)^2}{(4-\beta)^2})} \geq \frac{\beta(1-\beta)}{(4-\beta)^2}.
\]

Assume that \( \tilde{B}_{t+1,L} \geq \tilde{B}_{t+2,L} \), and we now show that \( \tilde{B}_{t,L} \geq \tilde{B}_{t+1,L} \). Using (13),

\[
\tilde{B}_{t,L} - \tilde{B}_{t+1,L} = \frac{(1-\beta)\beta^2(\beta - \alpha(1-\beta)\tilde{B}_{t+1,L})}{(4\beta - \beta^2 - 4\alpha(1-\beta)\tilde{B}_{t+1,L})^2} - \frac{(1-\beta)\beta^2(\beta - \alpha(1-\beta)\tilde{B}_{t+2,L})}{(4\beta - \beta^2 - 4\alpha(1-\beta)\tilde{B}_{t+2,L})^2}.
\]
We can easily check that \( \frac{\beta - \alpha (1-\beta)x}{2(1-\beta)(\beta - \alpha (1-\beta)B_{t+1,L})} \) is strictly increasing in \( x \) when \( 4\beta - 4\alpha (1-\beta)x > 0 \). Therefore, \( \tilde{B}_{t+1,L} - \tilde{B}_{t+2,L} \geq 0 \) by the induction hypothesis, \( \tilde{B}_{t+1,L} \geq \tilde{B}_{t+2,L} \), and the fact that

\[
4\beta - 4\alpha (1-\beta)\tilde{B}_{t+2,L} > 0 \quad \text{always hold by inequality (44)}.
\]

Since \( \tilde{B}_{T,L} = \frac{\beta(1-\beta)}{4(1-\beta)^2} \), using (10), \( \tilde{A}_{T-1,H} = 2(1-\beta)\left(1 - \frac{\alpha (1-\beta)^2}{4(1-\beta)^2}\right) \geq 0 \). Therefore, \( \tilde{B}_{t+1,L} \geq \tilde{B}_{t+2,L} \), we thus prove that \( \tilde{A}_{t,H} - \tilde{A}_{t+1,H} \geq 0 \).

Similarly, we can check that \( \tilde{A}_{T-1,L} \geq \tilde{A}_{T,L} \), and for \( T - 2 \geq t \geq 1 \),

\[
\tilde{A}_{t,L} - \tilde{A}_{t+1,L} = \frac{\beta^2 (1-\beta)}{4\beta - 4\alpha (1-\beta)\tilde{B}_{t+1,L}} \geq 0.
\]

The inequality follows with \( \tilde{B}_{t+1,L} \geq \tilde{B}_{t+2,L} \).

Last, we show that \( \tilde{B}_{t+1,H} \geq \tilde{B}_{t+2,H} \) by induction. We can check that \( \tilde{B}_{T-1,H} \geq \tilde{B}_{T,H} \). Assume that \( \tilde{B}_{t+1,H} \geq \tilde{B}_{t+2,H} \), and we now show this implies that \( \tilde{B}_{t,H} \geq \tilde{B}_{t+1,H} \). Using (12),

\[
\frac{\tilde{B}_{t,H} - \tilde{B}_{t+1,H}}{(1-\beta)\left(4(\beta - \alpha (1-\beta)\tilde{B}_{t+1,L})^2 + \alpha (1-\beta)\beta^2 \tilde{B}_{t+1,H}\right)} \geq 0.
\]

The last inequality follows from the induction hypothesis \( \tilde{B}_{t+1,H} \geq \tilde{B}_{t+2,H} \) and the result \( \tilde{B}_{t+1,L} \geq \tilde{B}_{t+2,L} \), which we have demonstrated. ■

**Proof of Proposition 8**

We prove the proposition by constructing a solution to the optimization problem (16)–(19) faced by firm L. This is done through dynamic programming. First consider the problem in the last period. Suppose the remaining customers have valuations \([0, \theta_T]\) \( (\theta_T \geq 0) \). Firm L’s problem is given by

\[
r_{T,L}(\theta_T) = \max_{\theta_T} \left\{ (p_{T,L} - \beta c) \left( \theta_T - \frac{p_{T,L}}{\beta} \right) \right\}.
\]

The optimal solution is determined by \( p^*_{T,L} = \frac{\beta}{2}(\theta_T - c) + \beta c \) and \( r^*_{T,L} = \frac{\beta}{4}(\theta_T - c)^2 \).
Assume in period \( t + 1 \) (\( t \geq 1 \)), given that the remaining customers have valuations \([0, \theta_{t+1}]\), firm L’s optimal price in period \( t + 1 \) and associated profit from period \( t + 1 \) to period \( T \) are given in (20) and (21). Now consider the problem in period \( t \):

\[
\begin{align*}
  r_{t,L}(\theta_t) &= \max_{p_{t,L}} \{(p_{t,L} - \beta c)(\theta_t - \theta_{t+1}) + \alpha r_{t+1}(\theta_{t+1})\}. \\
\end{align*}
\]

In the above, \( \theta_{t+1} \) is the valuation of a marginal customer who is indifferent between purchasing in period \( t \) and period \( t + 1 \) and therefore satisfies

\[
\beta \theta_{t+1} - p_{t,L} = \gamma (\beta \theta_t - p_{t+1,L}(\theta_t)) = \gamma (\beta \theta_t - C_{t+1,L}(\theta_t + c) - \beta c).
\]

Hence, \( \theta_{t+1} = \frac{p_{t,L} - \gamma c(\beta - C_{t+1})}{\beta - \gamma (\beta - C_{t+1})} \). Substituting into the profit function (45), we have

\[
\begin{align*}
  r_{t,L}(\theta_t) &= \max_{p_{t,L}} \left\{ (p_{t,L} - \beta c) \left( \theta_t - \frac{p_{t,L} - \gamma c(\beta - C_{t+1})}{\beta - \gamma (\beta - C_{t+1})} \right) + \frac{\alpha C_{t+1}}{2} \left( \frac{p_{t,L} - \beta c}{\beta - \gamma (\beta - C_{t+1})} \right)^2 \right\}. \\
\end{align*}
\]

We can easily check that the function within maximization is strictly concave and positive when \( \theta_t > c \). Therefore, the optimal price \( p_{t,L}^* \) is determined by:

\[
\begin{align*}
  p_{t,L}^*(\theta_t) &= \frac{(\beta - \gamma (\beta - C_{t+1}))^2}{2(\beta - \gamma (\beta - C_{t+1})) - \alpha C_{t+1}} (\theta_t - c) + \beta c = C_t(\theta_t - c) + \beta c. \\
\end{align*}
\]

The associated profit function \( r_{t,L}^* = \frac{1}{2} C_t(\theta_t - c)^2 \). Note that given \( p_{t,L} \), we have \( p_H^* = \frac{p_{t,L} + \beta c}{2(1 - \beta)} \) and \( \theta_t = \frac{1}{2} - \frac{p_{t,L} - \beta c}{2(1 - \beta)} \), according to (14) and (15). Hence, firm L’s profit over the entire \( T \) periods, \( r_{t,L}^* \), can be written as:

\[
\begin{align*}
  r_{t,L}^* &= \max_{p_{t,L}} \left\{ (p_{t,L} - \beta c) \left( \frac{1}{2} + \frac{c - p_{t,L}}{2(1 - \beta)} - \frac{p_{t,L} - \gamma c(\beta - C_2)}{\beta - \gamma (\beta - C_2)} \right) + \frac{\alpha C_2}{2} \left( \frac{p_{t,L} - \beta c}{\beta - \gamma (\beta - C_2)} \right)^2 \right\}. \\
\end{align*}
\]

This is a quadratic function in \( p_{t,L} \), and the optimal price \( p_{t,L}^* \) is then determined by the first-order condition as follows:

\[
\begin{align*}
  p_{t,L}^* &= \frac{\frac{1}{2}(1 - \beta)(1 - c)(\beta - \gamma (\beta - C_2))^2}{(\beta - \gamma (\beta - C_2))^2 + 2(1 - \beta)(\beta - \gamma (\beta - C_2)) - \alpha C_2(1 - \beta)} + \beta c \equiv C_t(1 - c) + \beta c. \\
\end{align*}
\]

The corresponding equilibrium profit \( r_{t,L}^* \) is \( \frac{1}{2} C_t(1 - c)^2 \). The optimal price of firm H and the associated profit are:

\[
\begin{align*}
  p_H^* &= \frac{p_{t,L}^* + \beta + c}{2} = \frac{1}{2}(1 - c)(1 - \beta + C_t) + c, \\
  r_H^* &= (p_H^* - c)(1 - \theta_t) = \frac{(1 - c)^2(1 - \beta + C_t)^2}{4(1 - \beta)}. \\
\end{align*}
\]

\[\blacksquare\]
Proof of Lemma 1

Given any price path of firm H, \( \{p_{i,t}\}_{t=1}^T \), define \( \hat{p}_L = \frac{\beta - \gamma}{1 - \gamma} p_{1,H} + \frac{\gamma (1 - \beta)}{1 - \gamma} p_{2,H} \). Because customers discount their utilities over time, \( \{p_{i,t}\}_{t=1}^T \) is monotonically decreasing in time. Note that

\[
\hat{p}_L > \frac{\beta - \gamma}{1 - \gamma} p_{2,H} + \frac{\gamma (1 - \beta)}{1 - \gamma} p_{2,H} = \beta p_{2,H} \geq \beta c.
\]

We prove the result by showing that if \( p_L < \hat{p}_L \), firm L incurs positive sales in period 1, while if \( p_L \geq \hat{p}_L \), firm L incurs no sales in the whole selling horizon.

When \( p_L < \hat{p}_L \), we have \( \frac{p_{1,H} - p_L}{1 - \beta} > \frac{\beta - \gamma}{\beta - \gamma} p_{2,H} \). This implies that there exists a non-empty set of \( \theta \) values satisfying \( \theta - p_{1,H} \geq \beta \theta - p_L \) and \( \beta \theta - p_L \geq \gamma (\theta - p_{2,H}) \). Meanwhile, note that taking \( t = 1 \) in (22) and (23) leads to \( \hat{\theta}_1 = \frac{p_{1,H} - p_{2,H}}{\beta - \gamma} \) and \( \hat{\theta}_2 = \frac{p_{1,H} - p_L}{1 - \beta} \), satisfying \( \hat{\theta}_1 < \hat{\theta}_2 \). Hence, there exists a feasible \( p_L \geq \beta c \) such that there is positive demand for firm L.

When \( p_L \geq \hat{p}_L \), we have \( \frac{p_{1,H} - p_{2,H}}{1 - \beta} \geq \frac{\beta - \gamma}{\beta - \gamma} p_{2,H} \). In this case, the threshold values \( \hat{\theta}_1 \geq \hat{\theta}_2 \), hence \( t \neq 1 \) in (22) and (23). This implies that customers with valuations greater than \( \frac{p_{1,H} - p_{2,H}}{1 - \beta} \) purchase product H in period 1, and all other customers would always be better off purchasing product H in period 2 than purchasing product L in period 1. This is because if customers buy product L, their valuations will satisfy \( \beta \theta - p_L \geq \gamma (\theta - p_{2,H}) \); that is, \( \theta \geq \frac{p_{1,H} - p_{2,H}}{\beta - \gamma} \) (note \( \beta > \gamma \)). However, since \( \frac{p_{1,H} - p_{2,H}}{\beta - \gamma} \geq \frac{p_{1,H} - p_{2,H}}{1 - \gamma} \), those customers would have purchased product H in period 1! Therefore, there is no sales for firm L over the entire selling horizon when it sets price greater than \( \hat{p}_L \). Therefore, for any given price path \( \{p_{i,t}\}_{t=1}^T \), firm L should price lower than \( \hat{p}_L \) such that \( t = 1 \) in (22) and (23).

Proof of Proposition 9

Since firm L only has positive demand in the first period, the game after period 3 is effectively a monopoly for firm H. Suppose \( T \geq 3 \). We first consider the last period. Given remaining customers on \( [0, \theta_T] \), the payoff function of firm H is

\[
r_{T,H}(\theta_T, p_{T,H}) = (p_{T,H} - c)(\theta_T - p_{T,H}).
\]

It can be easily verified that the optimal price and corresponding revenue are given by

\[
p^*_{T,H}(\theta_T) = \frac{\theta_T - c}{2} + c, r^*_{T,H}(\theta_T) = \frac{(\theta_T - c)^2}{4}.
\]

Therefore (27) and (28) hold for period \( T \). For \( t \geq 3 \), suppose (27), (28) and (32) hold for period \( t + 1 \). The valuation \( \hat{\theta} \) of a marginal customer who is indifferent between buying in period \( t \) and period \( t + 1 \) satisfies

\[
\hat{\theta} - p_{t,H} = \gamma (\hat{\theta} - p^*_{t+1,H}(\hat{\theta})) = \gamma (\hat{\theta} - D_{t+1}(\hat{\theta} - c) - c).
\]
It follows that \( \hat{\theta} = \frac{p_{t+1} + \gamma D_{t+1} c - c}{1 - \gamma + \gamma D_{t+1}} \). The payoff function in period \( t \) is given by

\[
r_{t,H}(\theta_t, p_{t,H}) = (p_{t,H} - c)(\theta_t - \hat{\theta}) + \frac{\alpha D_{t+1}(\hat{\theta} - c)^2}{2}.
\]

It can be verified that the solution in period \( t \) is given by (27) and (28) with \( D_t \) given by (32).

Suppose the price of \( L \) is \( p_L \). We next consider the problem in period 2. The valuation of the marginal customer who is indifferent between purchasing \( L \) in the first period and purchasing \( H \) in the second period is given by \( \frac{p_{L} - p_{2,H}}{\beta - \gamma} \). Similarly, the valuation of the marginal customer who is indifferent between purchasing \( H \) in the second and third periods is given by \( \frac{p_{2,H} + \gamma D_3 c - c}{1 - \gamma + \gamma D_3} \). It follows that the payoff function of \( H \) is given by

\[
r_{2,H}(p_L, p_{2,H}) = (p_{2,H} - c) \left( \frac{p_L - \gamma p_{2,H}}{\beta - \gamma} - \frac{p_{2,H} + \gamma D_3 c - c}{1 - \gamma + \gamma D_3} \right) + \frac{\alpha D_3}{2} \left( \frac{p_{2,H} + \gamma D_3 c - c}{1 - \gamma + \gamma D_3} - c \right)^2
\]

From the first-order condition, the optimal price for \( H \) in the second period can be written as a function of \( p_L \) as

\[
p_{2,H}^*(p_L) = T_4 p_L + T_5.
\]

The corresponding optimal revenue is given by

\[
r_{2,H}^*(p_L) = r_{2,H}(p_L, p_{2,H}^*(p_L)).
\]

The payoff functions for \( H \) and \( L \) in the first period are given by

\[
r_{1,H}(p_{1,H}, p_L) = (p_{1,H} - c) \left( 1 - \frac{p_{1,H} - p_L}{1 - \beta} \right) + r_{2,H}^*(p_L),
\]

\[
r_{L}(p_{1,H}, p_L) = (p_L - \beta c) \left( \frac{p_{1,H} - p_L}{1 - \beta} - \frac{p_L - \gamma p_{2,H}^*(p_L)}{\beta - \gamma} \right).
\]

Solving \( \frac{\partial r_{1,H}(p_{1,H}, p_L)}{\partial p_{1,H}} = 0 \) and \( \frac{\partial r_{L}(p_{1,H}, p_L)}{\partial p_L} = 0 \) gives (24) and (25). Expressions for equilibrium revenues can be derived by plugging the equilibrium prices.