Discrete Bandwidth Allocation Considering Fairness and Transmission Load in Multicast Networks

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Abstract

As a promising solution to tackle the network heterogeneity in multicasting, layered multicast protocols such as Receiver-driven layered multicast (RLM) and Layered video multicast with retransmission (LVMR) have been proposed. This paper considers fairness as well as transmission load in the layered multicasting. Lexicographically fair bandwidth allocation among multicast receivers is considered under the constraint of minimum bandwidth requirement and the link capacity of the network. The problem of transmission load in the layer multicasting due to various user requirements is also examined by minimizing the number of layers.

The bandwidth allocation is formulated as a nonlinear integer programming problem. A dual objective tabu search is proposed to solve the fairness and transmission load problem in multicast networks. Outstanding performance is obtained by the proposed tabu search. When the fairness objective is considered, the solution gap from the optimal solution is less than 2% in problems with 50 virtual sessions. The complexity of the dual objective largely depends on the weighting factor of the two objectives. Even in tough cases the proposed tabu search provides excellent solution the gap of which is within 6% from the optimal solution.

Keywords: Multicast network; Fairness; Layerd transmission; Bandwidth allocation; Tabu search

1. Introduction

Multicasting provides an efficient way of transmitting data from a sender to a group of receivers. Instead of sending a separate copy of the data to each individual group member, a source node sends one stream of messages to any one segment of the network on which there is a subscriber. An underlying routing algorithm determines a multicast tree connecting the source and group members. Data generated by the source flows through the multicast tree, traversing each tree edge exactly once. As a result, multicast is more resource-efficient and is well suited to applications such as teleconferencing, video-on-demand (VOD) service, electronic newspapers, cyber education and medical images.

In a multicast network multiple sessions each with different group members share network resources simultaneously. Thus, it is ideal to provide a fair share of bandwidth to each session. This issue of inter-session fairness has been extensively studied in unicast networks. In case of multicast, the other notion of fairness, i.e., intra-session fairness has to be considered because of the network heterogeneity that is due to various networks connected to the Internet. Users having high bandwidth connectivity would prefer to receive higher rate and higher quality service, while users with low bandwidth connectivity would be satisfied with low rate service. Thus, multirate multicast technology is necessary for transmission in heterogeneous networks. Receiver-driven layered multicast (RLM) [10] and Layered video multicast with retransmission (LVMR) [11] are well known protocols for layered multicast that satisfies the multirate multicast. Source signal is encoded and presented to the network as a set of bit streams, called layers. Layers are so organized that the quality of reception is proportional to the number of layers received. The first layer provides basic information, and every other layer improves data quality.

In layered multicast, a multicast session requires more and more layers to transmit as each

receiver in the multicast group requires different bandwidth due to the network heterogeneity. Here, increased number of layers in a session results in high overheads for sender encoding, multicast address allocation and receiver decoding. To prevent the high overheads required for layered multicast a source needs to set the number of layers to transmit and assign bandwidth to each layer by organizing requirement by receivers [4].

In this paper, we are interested in multicast transmission that satisfies fair bandwidth allocation with lower overhead. The number of layers employed for receivers in each session needs to be minimized, while satisfying fairness among receivers.

The paper is organized as follows. Section 2 discusses fairness and transmission load in multicast networks. A nonlinear integer programming model is presented in Section 3 for the bandwidth allocation problem. A dual objective tabu search is developed to solve the fairness and transmission load problem in Section 4. Optimal solution for the bandwidth allocation is discussed in Section 5. Computational results and conclusion are presented in Section 6 and 7 respectively.

2. The Issue of Fairness and Transmission Load in Multicast Network

When a network has profound heterogeneity, the fairness must include characteristics of multirate multicast network. Each source of multicast session transmits data to all of its receivers at different rate. One of frequently used definitions of fairness in multi-rate multicast networks is lexicographically optimal fairness [3, 12]. Differently from the well-known maxmin [2, 7, 8, 9] fairness, the lexicographically optimal fair allocation always exists in discrete case [3]. A bandwidth allocation vector is lexicographically optimal, if its smallest component is the largest among the smallest components of all feasible bandwidth allocation vectors. Subject to this, it has largest second smallest component, and so on.



Figure 1. Network with 2 multicast sessions and 3 virtual sessions

As an example, consider the network in Figure 1. In the figure a virtual session is defined as a source and receiver pair of a session. Each virtual session may have different data quality even if the original content is the same as other virtual sessions. Session 1 consists of virtual session 1 (path 1-3-4) and virtual session 2 (path 1-3-5), while session 2 consists of virtual session 3 (path 2-3-5). The bandwidth of each link capacity is given in the figure. The max-min fair bandwidth vector for the virtual session 1, 2, and 3 of the network is (3, 2.5, 2.5). The fair bandwidth allocation is restricted by link (3, 5). When continuous allocation of bandwidth is allowed, the max-min fairness always exists. However, in layered transmission scheme, bandwidth is allocated in discrete fashion.

In the network of Figure 1, if bandwidth is allocated in discrete layers, the max-min fair allocation vector does not exist. However, a lexicographically fair optimal allocation exists and given by (3, 2, 3) or (3, 3, 2). Note that lexicographically optimal fair bandwidth allocation is NP-hard in case of discrete layer allocation [3].

We now consider transmission load of layered transmission scheme. In layered transmission a signal is encoded into a number of layers that can be incrementally combined to provide progressive refinement. Thus transmission increases load as receivers in the same session require different number of layers. In view of lexicographic fairness, two allocation vectors (3, 2, 3) and (3, 3, 2) are equally fair. However, in view of transmission load each receiver of session

1 requires different bandwidth layers in (3, 2, 3). In case of (3, 3, 2), the transmission load is reduced since two receivers of session 1 require same number of layers.

In this paper, we are interested in discrete fair bandwidth allocation with reduced transmission load in multicast network with multiple sessions. In the next section, we will discuss modeling of the bandwidth allocation problem.

3. Bandwidth Allocation Problem for Fairness and Transmission Load

Consider a network with J multicast sessions and I multicast virtual sessions. The traffic of each session is transmitted from a source to a set of destination nodes across a predetermined multicast tree. We call the source and destination pair of a session as a virtual session.

For a virtual session *i*, let x_i be the bandwidth allocated to the virtual session and u_i be the minimum bandwidth requirement, then we have

$$x_i \ge u_i \qquad i=1,\,\ldots,\,I.$$

Now, consider a link l in the network where a set of virtual sessions of session j is passing through. Let v(j,l) be a set of virtual sessions belonging to session j and traversing through link l. In the multicast tree actual bandwidth assigned to session j is determined by the maximum bandwidth among the virtual sessions. Thus, by letting y_{il} be the maximum, we have

$$y_{jl} = \max_{i \in v(j,l)} x_i$$
 $j = 1, ..., J, l = 1, ..., L.$

Also, note that total bandwidth assigned to all virtual sessions traversing through link *l* cannot exceed the link capacity. By letting s(l) be a set of sessions passing through link *l*, and c_l be the link capacity, we have

$$\sum_{j\in s(l)} y_{jl} \leq c_l \quad l=1, \ldots, L.$$

In addition to above constraints, we need to consider transmission load that depends on the

number of layers required for each session. Let z_{ib} be a binary variable that represents the bandwidth unit *b* allocated to virtual session *i*. If the allocated bandwidth units for virtual session *i* is *b*, then $z_{ib} = 1$. Otherwise, $z_{ib} = 0$. To compute the number of layers we also define a binary variable n_{jb} for a session *j* with allocated bandwidth *b*. If the bandwidth unit allocated to a virtual session *i* belonging to a session *j* is *b*, then $n_{jb} = 1$. Thus the number of layers in a session *j* is determined by $\sum_{b=1}^{B} n_{jb}$. By letting v(j) be a set of virtual sessions belonging to session *j*, we

have

$$\begin{split} &\sum_{b=1}^{B} z_{ib} = 1 \qquad i = 1, ..., I. \\ &z_{ib} \leq n_{jb} \qquad j = 1, ..., J, \quad b = 1, ..., B, \quad i \in v(j) \end{split}$$

Here, x_i which is the bandwidth allocated to virtual session *i*, can be represented by the indicator variable, z_{ib} as

$$x_i = \sum_{b=1}^{B} b z_{ib}$$
 $i = 1, ..., I.$

Now, our objective is twofold. First, we need to allocate bandwidth to each virtual session such that the allocation satisfies the lexicographically optimal fairness. Secondly, we need to minimize the number of layers used for each multicast session.

For the lexicographic optimal fairness we consider a nonincreasing convex function $1/x^p$ where p is a large integer. Since the lexicographic optimal fairness maximizes the minimum component among all feasible solutions and subject to the maximization, maximizes the second minimum, etc., we consider the following objective function for the lexicographical optimal fairness as in [12].

$$\operatorname{Min} \sum_{i=1}^{I} 1/x_i^{p}$$

To improve the fairness the above objective function has to give more credit to a virtual

session x_i with smaller value. For unit increase of bandwidth, the smaller the component x_i , the larger the improvement of the objective function value. If the minimum bandwidth is maximized, then the second minimum is maximized when p is sufficiently large.

In addition to the lexicographic optimal fairness, we consider transmission load caused by the number of different layers in each multicast session. As discussed earlier, the number of different layers for session *j* is given by $\sum_{b=1}^{B} n_{jb}$. Thus the total number of layers to be used for all multicast sessions is given by $\sum_{j=1}^{J} \sum_{b=1}^{B} n_{jb}$.

By combining two objective functions with a factor α , our bandwidth allocation problem is formulated as follows.

Minimize
$$\alpha \sum_{i=1}^{I} 1/x_i^p + (1-\alpha) \sum_{j=1}^{J} \sum_{b=1}^{B} n_{jb}$$

subject to:

$$x_i \geq u_i \qquad \qquad i=1,\ldots,I \tag{1}$$

$$y_{jl} = \max_{i \in v(j,l)} x_i$$
 $j = 1, ..., J, l = 1, ..., L$ (2)

$$\sum_{i \in s(l)} y_{jl} \le c_l \qquad l = 1, \dots, L$$
(3)

$$\sum_{b=1}^{b} z_{ib} = 1 \qquad i = 1, ..., I \tag{4}$$

$$z_{ib} \le n_{jb}$$
 $j = 1, ..., J, \ b = 1, ..., B, \ i \in v(j)$ (5)

$$x_{i} = \sum_{b=1}^{B} b z_{ib} \qquad i = 1, ..., I$$
(6)

 $0 \le \alpha \le 1$

 $x_i \ge 0$ and integers, z_{ib} , n_{jb} are binary variables

Considering the NP-hardness of the lexicographically optimal fair allocation problem [3], the proposed nonlinear integer programming problem may not be effectively solved by any conventional optimization techniques. We consider a tabu search as a promising solution procedure for the above bandwidth allocation problem. Tabu search is a powerful search heuristic that has been successfully applied to numerous combinatorial optimization problems [13]. At each step of the search, neighborhood of the current solution is explored and the best one is selected as a new solution. The search procedure does not stop even when no improvement is obtained. The best solution in the neighborhood is selected, even if it is worse than the current solution. This strategy allows the search to avoid local optima and to explore a larger fraction of the solution space.

4. A Dual Objective Tabu Search for the Bandwidth Allocation

Tabu search is a high level heuristic procedure for solving optimization problems, designed to guide other methods to escape the trap of local optimality. It uses flexible structured memory to permit search information to be exploited more thoroughly than by rigid memory systems and memory functions of varying time spans for intensifying and diversifying the search.

Intensification strategies utilize short-term memory function to integrate features or environments of good solutions as a basis for generating still better solutions. Such strategies focus on aggressively searching for a best solution within a strategically restricted region. A move remains tabu during a certain period (or tabu time size) to help aggressive search for better solutions. Diversification strategies, which typically employ a long-term memory function, redirect the search to unvisited regions of the solution space.



Figure 2. Proposed tabu search procedure

In this paper, we propose a dual objective tabu search that considers fairness and transmission load. In the primary tabu search a solution that minimizes the fairness objective $\sum_{i=1}^{I} 1/x_i^p$ is

investigated with constraints (1), (2), and (3) of the formulation in Section 3.

The secondary tabu search starts with the solution obtained by the primary search. Since the objective of the secondary tabu search is to reduce the number of layers $\sum_{j=1}^{J} \sum_{b=1}^{B} n_{jb}$ at the sacrifice of fairness. A weighting factor 1- α is applied to transmission load. Figure 2 shows the overview of the tabu search.

Both primary and secondary tabu search incorporate following three procedures as shown in Figure 2.

1) Initial solution

- 2) Intensification with Short-Term Memory Function
- 3) Diversification with Long-Term Memory Function

Each procedure is discussed both for the fairness (primary tabu search) and transmission load (secondary tabu search) in the multicast transmission networks.

4.1 Initial solution

Since a solution has to satisfy the minimum required bandwidth constraint, each virtual session x_i starts with the minimum required bandwidth u_i . To have an initial solution that satisfies link constraint (3) a virtual session with the minimum bandwidth is selected and increased by one unit. Tie is broken randomly. This process is continued until all virtual sessions are saturated with the link capacities. The initial solution for the secondary tabu search is the best solution obtained by the primary tabu search.

4.2 Intensification with Short-Term Memory

Two types of moves "drop move" and "add move" are considered for the primary and secondary tabu search. In the primary search, a drop move is performed by selecting a virtual session x_i with the largest bandwidth. Its bandwidth x_i is decreased by one unit. Then add moves are implemented. For the lexicographically fair allocation of the bandwidth an add move selects a virtual session with the minimum bandwidth. It then increases the minimum bandwidth by one

unit. This is because improvement of the fairness objective function
$$\sum_{i=1}^{l} 1/x_i^p$$
 is maximized by

the smallest x_i . Tie is broken randomly. The above add moves are continued until no virtual session can be selected by the link capacity constraint. Tabu restriction is applied to a virtual session in the tabu list to restrict reversed or repeated move within a specific tabu time iterations.

Intensification procedure is continued until the search has no improvement for *I_max* consecutive iterations.

For the secondary tabu search, drop and add moves are applied to minimize the number of layers. In drop move the search selects a virtual session with maximum bandwidth and decreases its bandwidth to the nearest unit in the same session. Add move selects a virtual session with the minimum bandwidth and increases its bandwidth to the smallest unit higher than its current unit in the same session. Add moves are continued until no virtual session can be selected by the link capacity constraint.

4.3. Diversification with Long-Term Memory

Diversification strategy is helpful to explore new unvisited regions of the solution space. It enables the search process to escape from local optimality. The diversification is performed when the intensification process has no solution improvement for I_max consecutive iterations. This diversification strategy has the effect of restarting the tabu search from a solution that is far away from the solutions obtained in the intensification procedure. In the diversification procedure, a fraction of virtual sessions are selected and the bandwidth of each virtual session is increased or decreased depending on the historical frequencies of add and drop moves. The diversification procedure applied for the bandwidth allocation is as follows.

Step 1. For each virtual session, examine the frequency of add and drop moves applied.

- Step 2. Order the frequency from the minimum to the maximum and select a fraction of virtual sessions starting from the minimum frequency.
- Step 3. For each virtual session selected if the historical number of add moves is larger than drop moves, then decrease the bandwidth by one unit. Otherwise, increase the bandwidth by one unit.

5. Optimal Solution of the Bandwidth Allocation

In this section, we discuss the optimal solution of the dual objective bandwidth allocation problem. The bandwidth allocation problem proposed in Section 3 is a nonlinear integer problem. It is the nonlinear term $\sum_{i=1}^{l} 1/x_i^p$ that makes the problem hard to attack. However, the nonlinear function $1/x_i^p$ can be converted into a piecewise linear function $g(x_i)$ which is specified by points $(1, 1/1^p)$, $(2, 1/2^p)$, $(3, 1/3^p)$, ..., and $(B, 1/B^p)$, where *B* is the maximum available bandwidth units. Let $q_b(x)$ be a linear equation to connect $(b, 1/b^p)$ and $(b+1, 1/(b+1)^p)$. Then, $g(x_i)$ is a piecewise linear convex function of the form $\max_{b=1,...,B} q_b(x)$ [14]. Now by substituting the nonlinear term $\sum_{i=1}^{l} 1/x_i^p$ with $\sum_{i=1}^{l} g(x_i)$ and adding constraint $g(x_i) = \max_{b=1,...,B} q_b(x_i)$ for i = 1, ..., I, the formulation given in Section 3 can be converted into a linear

integer programming problem.

In Section 6, we will compare the performance of the proposed tabu search with the optimal solution by CPLEX [1] which is a well known branch and bound procedure.

6. Computational Results

In this section, we discuss the computational results of the Tabu Search for the bandwidth allocation. Three different sizes of multicast networks are generated as in Table 1. In each multicast network ten problems are tested with different link capacities. All solution procedures are run on a Pentium III-500MHz PC.

Before solving the bandwidth allocation problem we test the performance of tabu parameters: the tabu time size and I_max for the intensification procedure, the fraction of virtual sessions for diversification, and the stopping rule D_max of the diversification.

Number of links	Number of sessions	Number of virtual sessions	Minimum requirement of each virtual session
10	3	10	1~3
20	6	30	1~3
30	9	50	1~3

Table 1. Multicast networks

The number of	virtual sessions	10	30	50
Tabu	Primary	2	5	7
time size	me size Secondary		4	6

Table 2. Tabu time size of each problem

Procedure		Primary		Secondary			
Diversification	1/2	1/2	1/4	1/2	1/2	1 / 4	
fraction	1/2	1/3	1/3 1/4		1/3	1/4	
Objective	1.042	1.012	1 009	2 415	2 2 9 4	2 401	
function value	1.943 Inction value		1.912 1.998		2.384	2.401	

Table 3. Fraction of virtual sessions for diversification

Our test shows that the tabu time size is dependent on the size of virtual sessions. Larger tabu time size shows better performance as the problem size increases. However, the effect of tabu size is not that critical in problems with same number of virtual sessions. Tabu size with $10\sim20\%$ of the number of virtual sessions provides slightly better solutions. Appropriate tabu time size is shown in Table 2 for each problem size.

The test of I_max is performed with 30 virtual session problems as in Figure 3. By assuming that an appropriate value of I_max is proportional to the number of virtual sessions I, test is performed with five different values in primary and secondary tabu search. Figure 3 shows that $I_max = 5.2I$ is appropriate for the primary tabu search and $I_max = 4.0I$ for the secondary.



(a) primary tabu search



(b) secondary tabu search

Figure 3. Test of *I_max* with 30 virtual sessions



Figure 4. Test of *D_max* with 30 virtual sessions

Fraction of virtual sessions for diversification and the maximum number of diversification in tabu search are deeply related to the solution quality. Experiments with 30 virtual sessions are shown in Table 3. For both the primary and secondary tabu search, 1/3 of virtual sessions is appropriate to be selected for diversification. The test on D_max is also performed with 30 virtual sessions. 50 problems are experimented to determine D_max . Among 50 problems the portion that gives no further improvement is plotted after successive diversification in Figure 4. From the figure, it seems to be reasonable to perform six diversifications for the primary and four for the secondary tabu search. Longer diversification phase for the primary tabu search implies more difficulty of handling the fairness objective. In fact the nonlinear term in the objective function with large number of virtual sessions must be hard to attack compared to the linear term with small number of sessions. Based on the preliminary test for the proposed tabu parameters we now investigate the performance of tabu search for the bandwidth allocation problems.

6.1 Bandwidth Allocation for the Fairness Objective ($\alpha = 1$)

We first examine the fairness performance with the proposed primary tabu search. The genetic algorithm [12] and CPLEX [1] are also employed to compare solutions.

Table 4 shows the result of the primary fairness objective with 30 virtual sessions. As shown in the table, the proposed tabu search gives better solution quality compared to the genetic algorithm [12]. Optimal solutions are obtained by the proposed tabu search in all cases except for the problem 4 and 8.

Experiments are also performed for problems with 10 and 50 virtual sessions. Note that the number of virtual sessions is dependent on the number of multicast sessions in the network and the number of receivers in each session. Thus, 10, 30, and 50 virtual sessions are practical instances with the number of sessions given in Table 1. The proposed tabu search generated

optimal solutions in all cases with 10 virtual sessions, and three cases with 50 virtual sessions. Solution gap from the optimal solution is less than 2% in problems with 50 virtual sessions. GA also presents optimal solutions in all cases with 10 virtual sessions. However, the gap in the worst case reaches 7% in problems with 50 virtual sessions even with increased CPU times. CPU seconds by three search methods are compared as in Figure 5.

6.2 Bandwidth Allocation for the Dual Objectives ($\alpha < 1$)

Performance of the proposed tabu search is experimented with two objectives: fairness and transmission load. Table 5, 6 and 7 respectively show the fairness and number of layers employed for transmission in 10, 30 and 50 virtual sessions. In case of 10 and 30 virtual sessions, since the fairness value is much smaller than the number of layers, solutions are compared with $\alpha = 0.9$. In problems with 10 virtual sessions, proposed tabu search always gives optimal solutions as shown in Table 5. In problems with 30 virtual sessions, the transmission load is reduced when $\alpha = 0.9$ compared to the case of $\alpha = 1$. Number of layers presented by the tabu search exceeds that by the optimal solution by one unit in the worst cases. The fairness objective value by the tabu search exceeds 3% of optimal solutions obtained by the CPLEX.

The bandwidth allocation with 50 virtual sessions is shown in Table 7. The table shows that the fairness among virtual sessions is relaxed with reduced transmission load as α decreases. In case of $\alpha = 1$ the proposed tabu search solves only the fairness problem. It thus presents relatively higher number of layers compared to the optimal solution as shown in problem 4 and 9. However, the transmission load is dramatically reduced and converges to the optimal solution with the dual objective tabu search when $\alpha < 1$. The solution gap of the combined objective values by the tabu search is within 6% even in tough cases with $\alpha = 0.9$ and 0.8. Finally, Figure 6 compares CPU times by proposed tabu search and CPLEX [1]. The figure demonstrates that the proposed tabu search is time efficient compared to the CPLEX. The efficiency is critical in

Ducklass Duccedure				CPU
Problem	Procedure	Solution vector	value	seconds
	Tabu search	(3,3,3,3,4,4,4,4,4,4,4,5,5,5,5,5,5,5,5,5,	1.518	0.943
1	GA	(2,3,3,4,4,4,4,4,4,4,4,5,5,5,5,5,5,5,5,5,5	1.608	12.823
	CPLEX	(3,3,3,3,4,4,4,4,4,4,4,5,5,5,5,5,5,5,5,5,	1.518	1.631
	Tabu search	(3,3,4,4,4,4,4,4,4,4,4,4,5,5,5,5,5,5,5,5,	1.565	1.142
2	GA	(3,3,4,4,4,4,4,4,4,4,4,4,5,5,5,5,5,5,5,5,	1.565	10.326
	CPLEX	(3,3,4,4,4,4,4,4,4,4,4,4,5,5,5,5,5,5,5,5,	1.565	1.502
	Tabu search	(2,3,3,3,3,3,3,3,4,4,4,4,5,5,5,5,5,5,5,5,5	1.892	1.087
3	GA	(2,3,3,3,3,3,3,3,4,4,4,4,5,5,5,5,5,5,5,5,5	1.892	9.213
	CPLEX	(2,3,3,3,3,3,3,3,4,4,4,4,5,5,5,5,5,5,5,5,5	1.892	1.591
	Tabu search	(3,3,3,3,4,4,4,4,4,4,5,5,5,5,5,5,5,5,5,5,	1.436	1.362
4	GA	(3,3,3,3,3,3,4,4,4,4,5,5,5,5,5,5,5,5,5,6,6,6,6,6,6,8,11,12,12,14)	1.520	9.723
	CPLEX	(3,3,3,3,4,4,4,4,4,4,5,5,5,5,5,5,5,5,5,5,	1.434	1.634
	Tabu search	(3,3,3,3,3,3,3,4,4,4,4,5,5,5,5,5,6,6,6,6,6,7,7,7,8,8,8,12,12,16)	1.492	0.998
5	GA	(3,3,3,3,3,3,3,3,4,4,5,5,5,5,5,6,6,6,6,7,7,7,7,8,8,9,12,12,16)	1.579	12.423
	CPLEX	(3,3,3,3,3,3,3,4,4,4,4,5,5,5,5,5,6,6,6,6,6,7,7,7,8,8,8,12,12,16)	1.492	1.623
	Tabu search	(2,3,3,3,3,3,4,4,4,5,5,5,5,5,6,6,6,6,6,6,6,6,6,6,6,8,8,8,11,14,15)	1.547	1.202
6	GA	(2,3,3,3,3,3,3,4,4,5,5,5,5,5,6,6,6,6,6,6,6,6,6,7,7,8,8,8,11,14,15)	1.581	12.592
	CPLEX	(2,3,3,3,3,3,4,4,4,5,5,5,5,5,6,6,6,6,6,6,6,6,6,6,6,8,8,8,11,14,15)	1.547	1.603
	Tabu search	(3,3,3,3,3,3,3,3,4,4,5,5,5,5,5,6,6,6,6,7,7,7,7,8,8,9,12,12,16)	1.579	1.332
7	GA	(3,3,3,3,3,3,3,3,4,4,5,5,5,5,5,6,6,6,6,7,7,7,7,8,8,9,12,12,16)	1.579	12.693
	CPLEX	(3,3,3,3,3,3,3,3,4,4,5,5,5,5,5,6,6,6,6,7,7,7,7,8,8,9,12,12,16)	1.579	1.511
	Tabu search	(2,3,3,3,3,3,3,4,4,4,4,5,5,5,6,6,6,6,7,7,7,8,8,8,9,9,9,10,11,12)	1.568	1.059
8	GA	(2,3,3,3,3,3,3,4,4,4,4,5,5,6,6,6,6,7,7,7,8,8,8,8,9,9,11,11,12)	1.592	8.485
	CPLEX	(2,3,3,3,3,3,3,4,4,4,5,5,5,5,6,6,6,6,7,7,7,8,8,8,8,9,9,10,11,12)	1.549	1.932
	Tabu search	(2,2,2,3,3,3,3,3,4,4,4,4,5,5,5,5,5,5,5,8,8,8,8,9,10,13,13,13,13)	2.077	1.101
9	GA	(2,2,2,3,3,3,3,3,4,4,4,4,5,5,5,5,5,5,5,8,8,8,8,9,10,13,13,13,13)	2.077	9.287
	CPLEX	(2,2,2,3,3,3,3,3,4,4,4,4,5,5,5,5,5,5,8,8,8,8,9,10,13,13,13,13)	2.077	1.887
	Tabu search	(2,2,3,3,3,3,3,3,4,4,4,4,4,4,5,5,5,5,5,5,5,5	2.019	1.293
10	GA	(2,2,3,3,3,3,3,3,4,4,4,4,4,4,5,5,5,5,5,5,5,5	2.019	10.309
	CPLEX	(2,2,3,3,3,3,3,3,4,4,4,4,4,4,5,5,5,5,5,5,5,5	2.019	1.409

Table 4. Bandwidth allocation for fairness objective with 30 virtual sessions



Figure 5. CPU seconds for the fairness objective



Figure 6. CPU seconds for the dual objectives with 50 virtual sessions

			α = 1.0		$\alpha = 0.9$				
Problem	Procedure	Fairness	# of	CPU	Fairness	# of	CPU		
		value	layers	seconds	value	layers	seconds		
1	Tabu search	0.239	5	0.156	0.314	3	0.303		
1	CPLEX	0.239	5	0.388	0.314	3	1.584		
2	Tabu search	0.241	4	0.143	0.265	3	0.312		
2	CPLEX	0.241	4	0.332	0.265	3	1.391		
2	Tabu search	0.241	5	0.165	0.263	3	0.283		
3	CPLEX	0.241	5	0.291	0.263	3	1.574		
4	Tabu search	0.269	7	0.101	0.400	3	0.232		
4	CPLEX	0.269	7	0.291	0.400	3	1.634		
5	Tabu search	0.189	4	0.089	0.193	3	0.216		
	CPLEX	0.189	4	0.292	0.193	3	1.483		
6	Tabu search	0.178	5	0.122	0.181	3	0.192		
0	CPLEX	0.178	5	0.313	0.181	3	1.531		
7	Tabu search	0.241	4	0.128	0.257	3	0.250		
/	CPLEX	0.241	4	0.311	0.257	3	1.618		
0	Tabu search	0.255	6	0.113	0.267	3	0.213		
8	CPLEX	0.255	6	0.302	0.267	3	1.574		
0	Tabu search	0.300	5	0.097	0.308	3	0.235		
9	CPLEX	0.300	5	0.298	0.308	3	1.616		
10	Tabu search	0.250	5	0.131	0.255	3	0.253		
10	CPLEX	0.250	5	0.221	0.255	3	1.544		

Table 5. Performance of tabu search with 10 virtual sessions

			α = 1.0				
Problem	Procedure	Fairness	# of	CPU	Fairness	# of	CPU
		value	layers	seconds	value	layers	seconds
1	Tabu search	1.518	10	0.943	1.632	7	2.023
1	CPLEX	1.518	10	1.631	1.632	7	7.158
2	Tabu search	1.565	11	1.142	1.726	6	2.481
2	CPLEX	1.565	11	1.502	1.678	6	7.540
2	Tabu search	1.892	10	1.087	2.042	8	2.337
3	CPLEX	1.892	10	1.591	2.003	7	7.923
	Tabu search	1.436	11	1.362	1.563	8	2.141
4	CPLEX	1.434	10	1.634	1.547	7	8.062
5	Tabu search	1.492	11	0.998	1.602	7	2.094
	CPLEX	1.492	11	1.623	1.598	7	7.050
6	Tabu search	1.547	10	1.202	1.653	6	2.442
0	CPLEX	1.547	10	1.603	1.653	6	6.874
7	Tabu search	1.579	10	1.332	1.597	6	2.442
/	CPLEX	1.579	10	1.511	1.597	6	7.031
0	Tabu search	1.568	11	1.059	1.583	7	2.363
0	CPLEX	1.549	10	1.932	1.556	6	7.242
0	Tabu search	2.077	12	1.101	2.093	6	2.545
7	CPLEX	2.077	12	1.887	2.093	6	7.652
10	Tabu search	2.019	10	1.293	2.043	7	2.511
10	CPLEX	2.019	10	1.409	2.031	7	8.123

Table 6. Performance of tabu search with 30 virtual sessions

		α = 1.0			$\alpha = 0.9$			$\alpha = 0.8$			$\alpha = 0.7$		
Problem	Procedure	Fairness	# of	CPU	Fairness	# of	CPU	Fairness	# of	CPU	Fairness	# of	CPU
		value	layers	seconds	value	layers	seconds	value	layers	seconds	value	layers	seconds
1	Tabu search	2.361	28	3.2	2.752	17	7.8	3.413	13	6.8	4.736	11	5.3
1	CPLEX	2.352	26	4.1	2.743	15	113.8	3.321	12	100.4	4.583	10	51.7
n	Tabu search	3.293	26	3.8	3.712	16	5.7	4.298	12	6.1	6.104	11	5.3
2	CPLEX	3.282	27	5.1	3.565	15	129.2	4.162	11	107.3	5.972	10	52.8
2	Tabu search	2.975	30	3.3	3.513	17	6.7	4.129	12	5.9	5.629	10	4.7
3	CPLEX	2.975	30	3.7	3.472	15	115.9	4.032	11	102.6	5.521	10	55.3
4	Tabu search	2.475	29	4.5	2.789	18	7.3	3.376	11	6.7	4.826	10	5.1
4	CPLEX	2.464	25	5.0	2.784	17	103.5	3.246	11	99.8	4.826	10	49.8
5	Tabu search	2.611	31	3.6	3.031	17	7.1	3.620	13	6.9	5.035	11	5.8
5	CPLEX	2.611	31	4.7	2.976	15	101.8	3.542	12	109.7	5.035	11	47.6
6	Tabu search	2.425	30	3.9	2.654	15	6.5	3.213	12	6.2	4.472	11	4.8
0	CPLEX	2.422	30	5.9	2.654	15	110.3	3.213	12	121.6	4.340	10	55.7
7	Tabu search	2.184	29	3.2	2.610	16	6.8	3.298	12	5.8	3.854	11	8.3
/	CPLEX	2.172	28	5.3	2.532	15	107.2	3.172	12	115.3	3.854	11	51.3
o	Tabu search	2.385	27	2.6	2.853	16	7.3	3.194	13	6.2	4.593	11	5.3
8	CPLEX	2.373	26	3.2	2.752	16	114.2	3.105	12	121.7	4.583	10	53.2
0	Tabu search	2.558	30	3.6	2.866	17	7.1	3.509	13	6.9	4.240	11	6.2
9	CPLEX	2.521	25	4.9	2.863	15	120.3	3.427	12	112.7	4.186	10	52.1
10	Tabu search	2.633	28	4.2	3.073	18	5.9	3.417	14	5.7	4.386	10	5.1
10	CPLEX	2.633	28	7.0	2.984	17	109.8	3.532	13	109.5	4.386	10	48.1

Table 7. Performance of tabu search with 50 virtual sessions

real applications when multicast members frequently leave and join their group. In this case each source of multicast session needs to periodically update the bandwidth for its receivers, which must be a burden to the network when the size of the problem increases.

7. Conclusion

Bandwidth allocation problem in multicast networks is examined by taking fairness and transmission load into account. Fairness is related to the discrete number of bandwidth units in layered transmission in multicast networks. Minimizing the number of layers employed in all session is also considered to reduce the transmission load.

The problem is formulated as a nonlinear integer programming that provides bandwidth allocation to each virtual session subject to the minimum bandwidth requirement, actual bandwidth assigned to each session in a link, and the link capacity constraint. The objective is to have lexicographically fair allocation among virtual sessions and to minimize the number of layers employed for all sessions.

A dual objective tabu search is proposed to solve the bandwidth allocation problem. It initially solves fairness then transmission load by slightly relaxing the fairness. Add and drop moves are employed to intensify the solution according to the short-term memory. Diversification by the historical frequency is implemented with the long-term memory.

Computational experiments are performed in multicast networks with 10, 30, and 50 virtual sessions. First, fairness performance is examined with the proposed primary tabu search. The proposed tabu search generates optimal solutions in all cases with 10 virtual sessions. In problems with 30 and 50 virtual sessions the solution gap from the optimal solution is less than 2%. The GA approach [12] is also examined. The performance, however, is not so desirable in large problems. In problems with 50 virtual sessions the solution gap in reaches up to 7% even

with increased CPU times. The effectiveness of proposed tabu search is demonstrated as the network size increases.

Secondly, the combined objective of the fairness and the transmission load is experimented. In problems with 10 and 30 virtual sessions outstanding performance is obtained with the proposed tabu search. The optimal transmission load is obtained in most cases except some cases in 30 virtual sessions. The excellence of the proposed tabu search is demonstrated in 50 virtual session problems. The transmission load is dramatically reduced and converges to the optimal even in tough cases of the dual objectives.

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