## Relative Building-Block fitness and the Building-Block Hypothesis

## Introduction

Relative Building-Block fitness and the
Building-Block Hypothesis
Written by Stephanie Forrest, Melanie Mitchell
Written on 1992

## Introduction-from class

## Schema Theorem

Short,
Low-order,
Highly-fit schemas(building blocks)
receive exponentially increasing trials in subsequent generations

## Introduction

## On what condition does GA perform well?

Identify features of fitness landscapes that are particularly relevant to the GA's performancebuilding blocks
Design simplified landscapes containing different configurations of such features-distribution, frequency, size
Study in detail the effects of these features on the GA's behaviorthe way in which schemas are processed and building blocks are combined

## Stepping-stones in the crossover landscape

Two landscape features of building-block hypothesis
presence of short, low-order, highly fit schemas the presence of intermediate "stepping stone"-intermediate-order higher-fitness schemas that result from combinations of the lower-order schemas => how much higher in fitness do the intermediate stepping stones have to be for the GA to work well?

## Stepping-stones in the crossover landscape

Royal Road functions
Select an optimum string and break it up into a number of small building blocks
Assign values to each low-order schema and each possible intermediate combination of low-order schemasuse the values to compute the fitness of a bit string

## Stepping-stones in the crossover landscape

## Royal Road functions-R1

```
s
s}\mp@subsup{s}{2}{}=********11111111***************************************************; c2 co
s3 = *****************11111111******************************************; c3 = 8
s
s
s6 = ******************************************111111111****************; c6 = 8
```



```
s8 = *************************************************************11111111; c% = 8
sopt =11111111111111111111111111111111111111111111111111111111111111111111
```

$\mathrm{R} 1(\mathrm{x})$ is computed by summing the coefficients c (order) of corresponding schema of which $x$ is an instance R1(1111111100....0)=8
R1(1111111100......011111111)=16
Can know how lower-level blocks are combined into higher-level blocks

## Stepping-stones in the crossover landscape

## Royal Road functions-R2- consider more on crossover

```
s
s}\mp@subsup{s}{2}{=
s
s}\mp@subsup{s}{4}{}=**************************111111111***********************************; c; c4 = 8
s
```



```
s
s
s9 = 1111111111111111****************************************************; c
```




```
s 12 =***************************************************1111111111111111; c12 = 16
```



```
s
sopt}=1111111111111111111111111111111111111111111111111111111111111111111111
R2(1111111100....0)=8
R2(1111111100\ldots....011111111)=16
R2(11111111111111110........0)=32
```

$\mathrm{R} 2(\mathrm{opt})=8 * 8+16 * 4+32 * 2=192$

## Royal Road experiments

Initial Variables
Length of string: 64
GA population size:128
Initial population generated randomly
GA continue until find optimum discovered-check total number of function evaluations performed
Single crossover: rate 0.7
Mutation : rate 0.005

## Royal Road experiments

Initial Variables
Reproduction: expected number of offspring

$$
1+\frac{F_{i}-\bar{F}}{2 \sigma}
$$

Maximum expected offspring of any string was 1.5 -if the
formula gives higher value, it was reset to 1.5
-most individuals will reproduce only $0,1,2$ times.
-to slow down convergence

## Royal Road experiments

Initial Variables
Randomly generated sting on bottom-level
Probability of having order-8 Schemas
$=8 * \frac{1}{2^{8}}=\frac{1}{32}$
Initial number of order- 8 schemas :128/32=4

## Royal Road experiments

Experiments on R1 and R2
R2 has more clear path by crossover(high fitness value for highest order schemas)
Has stronger path to optimum
Hypothesis: R2 will perform well than R1 finding optimum

## Royal Road experiments

Experiments on R1 and R2

| ORIGINAL EXPERIMENT |  |  |
| :--- | :--- | :--- |
|  | Function Evaluations to Optimum |  |
| 500 runs | $R 1$ | $R 2$ |
| Mean | 62099 (std err: 1390) | 73563 (std err: 1794) |
| Median | 56576 | 66304 |

Table 1: Summary of results of running the GA on $R 1$ and $R 2$. The table gives the mean and median function evaluations taken to find the optimum over 500 runs on each function. The numbers in parentheses are the standard errors.
$R 1$ is better than $R 2$ !

## Royal Road experiments

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## Potential bottleneck?

Deception?=> R2 is non deceptive function
$=>$ trace schemas through run

## Royal Road experiments


s1 and s2 combine quickly and has high density of the population
Remember small dip on 220

## Royal Road experiments


s4 rises fast, but s3 and s10 does not have population
on generation 120 and 535 it gets generation but it dies out s4 has a dip on time 220

## Roval Road experiments

Evolution of schemas 5, 6, and 11 (see Figure 2) - Slide 8

s6 appears around generation 30 and rise quickly.
s5 appears around generation 20 and disappears, and appears again on generation 220 , and grows quickly with s11
This rise coincide with the minor dip on $\mathrm{s} 1, \mathrm{~s} 2, \mathrm{~s} 9$, major dip on s 4

## Royal Road experiments

$s 9$ has high fitness value of 32 , it causes very quickly rise compared to 44 which has fitness value of 8
It tends out to push out existing instances of s4 in the population "Hitchhiking"
0 's in other positions in the string hitchhike along with the highly fit s11
Most likely positions for hitchhikers are those close to the highly fit schema's defined positions-cause by crossover
Power of crossover to combine lower-level building blocks was hampered, because it is get suppressed partially or totally by the quick rise of disjoint building blocks

## Royal Road experiments

For R1, which laks the extra fitness given to some intermediate-level schemas, hitchhiking problem does not occur to such a devasting degree-s11 fitness value is only 16
Contrary to hypothesis, extra reinforcement from some intermediatelevel stepping-stones actually harms the GA

## Royal Road experiments

## Change on variables

Hypothesis: The result is from lack of population-sampling error
Experiment1: GA with population size 1024
Experiment2: GA with lowest-order schemas length 4 instead of 8

## Royal Road experiments

| POPULATION SIZE 1024 |  |  |
| :--- | :--- | :--- |
|  | Function Evaluations to Optimum |  |
| 200 runs | $R 1$ | $R 2$ |
| Mean | 37453 (std err: 868 ) | 43213 (std err: 1275) |
| Median | 34816 | 36864 |

Table 2: Summary of results of 200 runs of the GA with population size 1024 on $R 1$ and $R 2$.

| LOWEST-ORDER SCHEMAS LENGTH 4 |  |  |
| :--- | :--- | :--- |
|  | Function Evaluations to Optimum |  |
| 200 runs | $R 1$ | $R 2$ |
| Mean | 6568 (std err: 198) | 11202 (std err: 394) |
| Median | 5760 | 9600 |

Table 3: Summary of results of 200 runs of the GA on modified versions of $R 1$ and $R 2$, in which the lowest-order building blocks are of length 4 .

Do not change the qualitative difference between R1 and R2

## Royal Road experiments

## Conclusion

We observe premature convergence even in very simple setting
The population loses useful schemas once one of the disjoint good schemas is found suggests one reason that the rate of effective implicit parallelism of GA may need to be reconsidered

## Royal Road experiments

## Using introns

Hitchhiking occurred in the loci that were spatially adjacent to the high-fitness schemas
Construct new function R2introns by introducing blocks of 8
"introns" between each of the 8 -bit blocks of 1 's
s1=11111111********
$\mathrm{S} 2=* * * * * * * * * * * * * * * * 11111111 * * * * * * * *$

Blocks are each separated so has least damage by hitchhiking

## Royal Road experiments

## Weaker nonlinear reinforcement

R1 has linear reinforcement, the fitness of an instance of an intermediate-order schema is always the sum of the fitness of instances of the component blocks
R2 has nonlinear reinforcement, the fitness goes much higher
=> weaker nonlinear reinforcement function : R2flat
c1-c14 is set to 1 , so s 9 has fitness value $3, \mathrm{~s} 1, \mathrm{~s} 2=1$ it is still nonlinear, but more flatten then R2

## Royal Road experiments

| ORIGINAL EXPERIMENT |  |  |
| :--- | :--- | :--- |
|  | Function Evaluations to Optimum |  |
| 500 runs | $R 1$ | $R 2$ |
| Mean | 62099 (std err: 1390) | 73563 (std err: 1794) |
| Median | 56576 | 66304 |

Table 1: Summary of results of running the GA on $R 1$ and $R 2$.

| VARIANTS OF R2 |  |  |
| :--- | :--- | :--- |
|  | Function Evaluations to Optimum |  |
| 200 runs | $R 2_{\text {introns }}$ | $R 2_{\text {flat }}$ |
| Mean | 75599 (std err: 2697) | 62692 (std err: 2391) |
| Median | 70400 | 56448 |

Table 4: Summary of results of 200 runs of the GA on two variants of $R 2$.

## R2introns vs R2 / R2flat vs R1

## Royal Road experiments

R2introns vs R2
-similar with R2 (no advance)
-convergence is so fast that hitchhikers are possible even
in loci that are relatively distant from the schema's defined position
R2flat vs R1
-average time approximately same with R1
-no advance, but no hurt on performance

## Experiments with hill-climbing

Steepest-ascent hill-climbing
Next-ascent hill-climbing
Random-mutation hill-climbing


## Experiments with hill-climbing

- Steepest-ascent hill-climbing (SAHC):

1. Choose a string at random. Call this string current-hilltop.
2. Systematically mutate each bit in the string from left to right, recording the fitnesses of the resulting strings.
3. If any of the resulting strings give a fitness increase, then set current-hilltop to the resulting string giving the highest fitness increase.
4. If there is no fitness increase, then save current-hilltop and go to step 1 . Otherwise, go to step 2 with the new current-hilltop.
5 . When a set number of function evaluations has been performed, return the highest hilltop that was found.

## Experiments with hill-climbing

- Next-ascent hill-climbing (NAHC):

1. Choose a string at random. Call this string current-hilltop.
2. Mutate single bits in the string from left to right, recording the fitnesses of the resulting strings. If any increase in fitness is found, then set currenthilltop to that increased-fitness string, without evaluating any more singlebit mutations of the original string. Go to step 2 with the new currenthilltop, but continue mutating the new string starting after the bit position at which the previous fitness increase was found.
3. If no increases in fitness were found, save current-hilltop and go to step 1.
4. When a set number of function evaluations has been performed, return the highest hilltop that was found.

## Experiments with hill-climbing

- Random-mutation hill-climbing (RMHC):

1. Choose a string at random. Call this string best-evaluated.
2. Choose a locus at random to mutate. If the mutation leads to an equal or higher fitness, then set best-evaluated to the resulting string.
3. Go to step 2.
4. When a set number of function evaluations has been performed, return the current value of best-evaluated.

## Experiments with hill-climbing

| HILL-CLIMBING ON R2 |  |  |  |
| :--- | :--- | :--- | :--- |
|  | Function Evaluations to Optimum |  |  |
| 200 runs | SAHC | NAHC | RMHC |
| Mean | $>256,000$ (std err: 0) | $>256,000$ (std err: 0) | 6551 (std err: 212) |
| Median | $>256,000$ | $>256,000$ | 5925 |

Table 5: Summary of results of 200 runs of various hill-climbing algorithms on $R 2$.
GA performs better than SAHC, NAHC-they didn't get even optimum
RMHC has average 10 times faster than GA on population size 128 , 6 times faster than size 1024
-it is ideal for the Royal Road functions, but will have trouble on function with local minima

## Experiments with hill-climbing <br> Online performance



Figure 4: Plots of the average on-line performance of the GA (population sizes 128 and 1024) and of random-mutation hill-climbing (RMHC), over 100 runs. The plot for RMHC stops at around 6000 function evaluations because RMHC had almost always found the function optimum by that time.

## Conclusions

Can understand more precisely how schemas are processed under crossover.
Hitchhiking is evidently one bottleneck for GA
Can improve this by adding noise, including all
combinations of lower-order schemes in the explicit list of schemas, allowing schemas to overlap

## +GA compared with Hill climbing on TSP

Comparison of Genetic Algorithm and Hill Climbing for Shortest Path Optimization Mapping- [Mona Fronita, Rahmat Gernowo , and Vincencius Gunawan]


Fig. 4 Distance testing 8 cities


Fig. 5 Distance testing 16 cities

## +GA compared with Hill climbing on TSP

Comparison of Genetic Algorithm and Hill Climbing for Shortest Path Optimization Mapping- [Mona Fronita, Rahmat Gernowo , and Vincencius Gunawan]


Fig. 6 Distance testing 24 cities


Fig. 7 Distance testing 32 cities

Thank you for listening!

